All India Test Series (JEE–2024)

AVJLM1/06

Test- 06

Lakshya JEE 2024

DURATION : 180 Minutes

DATE : 16/01/2024

M. MARKS : 300

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(3)	31.	(1)	61.	(2)
2.	(4)	32.	(2)	62.	(3)
3.	(2)	33.	(2)	63.	(1)
4.	(3)	34.	(3)	64.	(3)
5.	(2)	35.	(3)	65.	(2)
6.	(4)	36.	(1)	66.	(3)
7.	(2)	37.	(4)	67.	(2)
8.	(1)	38.	(2)	68.	(1)
9.	(2)	39.	(3)	69.	(3)
10.	(3)	40.	(4)	70.	(3)
11.	(4)	41.	(2)	71.	(4)
12.	(1)	42.	(2)	72.	(1)
13.	(2)	43.	(1)	73.	(1)
14.	(1)	44.	(4)	74.	(1)
15.	(1)	45.	(3)	75.	(3)
16.	(4)	46.	(1)	76.	(2)
17.	(2)	47.	(2)	77.	(1)
18.	(1)	48.	(1)	78.	(3)
19.	(1)	49.	(1)	79.	(2)
20.	(2)	50.	(3)	80.	(4)
21.	(1)	51.	(4)	81.	(2)
22.	(15)	52.	(5)	82.	(16)
23.	(5)	53.	(5)	83.	(2)
24.	(30)	54.	(87)	84.	(1)
25.	(3)	55.	(400)	85.	(63)
26.	(8)	56.	(5)	86.	(4)
27.	(9)	57.	(285)	87.	(5)
28.	(4)	58.	(5)	88.	(4)
29.	(3)	59.	(5)	89.	(6)
30.	(2)	60.	(4)	90.	(2)

SECTION-I (PHYSICS)

1. **(3)**

$$\rho = \frac{3.14\times10^{-6}\times5}{1}\text{ Ohm Metre}$$

$$= 15.7\mu\text{ Ohm Metre}$$

$$\delta\rho = \rho\left[\frac{2\delta r}{r}+\frac{\delta L}{L}+\frac{\delta R}{R}\right]\text{ Ohm Metre}$$

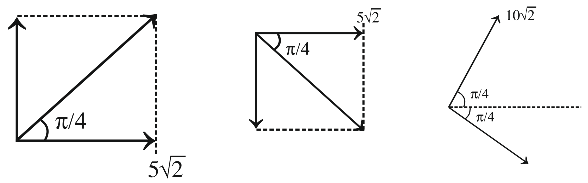
$$= 15.7\mu\rho\left[2\times\frac{.01}{1}+\frac{.01}{1}+\frac{.01}{5}\right]\text{ Ohm Metre}$$

$$= 15.7\mu[.02+.01+.002]\text{ Ohm Metre}$$

$$= 15.7\times.032\mu\Omega\,\text{m}$$

$$= .50\ \mu\Omega\,\text{m}$$

2. **(4)**

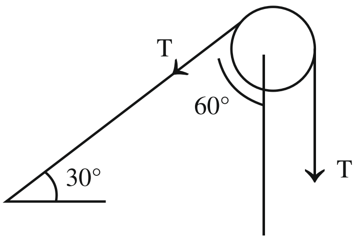


$$I_c = \frac{100}{5\sqrt{2}}\sin\left(50t+\frac{\pi}{4}\right)\ \text{A}$$

$$I_l = \frac{100}{5\sqrt{2}}\sin\left(50t-\frac{\pi}{4}\right)\ \text{A}$$

$$I = 20\sin(50t)\ \text{A}$$

3. **(2)**



$2mg\sin30 - T = 2m(a)$

$T = ma$

$$\Rightarrow a = g/3 \quad \& \quad T = \frac{mg}{3}$$

$\therefore$ Net force =

Resultant of these forces =

$$= T\left(-\hat{j}\right)+T\cos60^\circ\left(-\hat{j}\right)+T\sin60(-\hat{i})$$

4. **(3)**

In steady state :-

$$V_P - V_Q = \frac{E}{2R}\times R = \frac{E}{2}$$

$$V_Q - V_S = \frac{E}{2R}\times R = \frac{E}{2}$$

$$V_S = V_R$$

$$V_P - V_R = E$$

$$\therefore Q_A = CE$$

$$Q_B = \frac{CE}{2}$$

5. **(2)**

$V_P - V_0 = (10\times10+10\times10)$

= -200 volts

6. **(4)**

$$\Delta x = 2\text{ cm}, \Delta\phi = k\Delta x = 100\times2 = 200\,\text{rad}$$

7. **(2)**

$$P_0 - \frac{2T}{r}+\rho gh = P_0 - \frac{2T}{2r}$$

$$\Rightarrow T = \rho ghr$$

8. **(1)**

$$S_A = v_0t - \frac{1}{2}\mu_A gt^2$$

$$S_B = v_0t - \frac{1}{2}\mu_B gt^2$$

$$S_A - S_B = L - l \quad \Rightarrow t = \sqrt{\frac{2(L-l)}{(\mu_B-\mu_A)g}}$$

9. **(2)**

As tube is fixed, ball will come out symmetrically.

10. **(3)**

If E= electrical field due to each quadrant,

then $E_x = \frac{E}{\sqrt{2}},\ E_y = \frac{3E}{\sqrt{2}}$; $\tan\theta = \frac{1}{3}$

11. **(4)**

$$v_p = \frac{GM}{r} = -\frac{GM}{\sqrt{5}R};\ v_0 = -\frac{GM}{R}$$

From conservation of energy,

$$u_i + k_i = u_f + k_f$$

$$\Rightarrow -\frac{GMm}{\sqrt{5}R}+0 = -\frac{GMm}{R}+\frac{1}{2}mv^2$$

where m = mass of particle

$$\Rightarrow v = \sqrt{\frac{2GM}{R}\left(1-\frac{1}{\sqrt{5}}\right)}$$

12. **(1)**

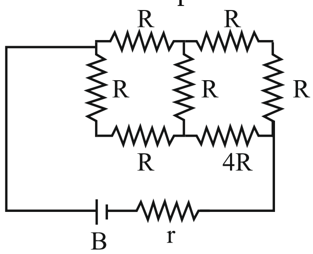
Facing surface of block will have opposite charge.

As a result attractive force will develop.

13. **(2)**

Circuit is forming a balanced wheatstone bridge. $R_{eq} = 2R$

For maximum power transfer $2R = r$.



14. **(1)**

$$\phi = \frac{\sqrt{3}}{4} l^2 B$$

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\sqrt{3}}{4} l^2 \frac{dB}{dt}$$

$$i = \frac{\varepsilon}{R} = \frac{\sqrt{3} l^2}{4R}$$

15. **(1)**

$V = Es$;

$$S = 0 + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$E = \frac{2sm}{qt^2}$$

$$\Rightarrow v = \frac{2s^2 m}{qt^2} = 11.375 \text{ Volt}$$

16. **(4)**

Static friction depends on tendency of relative velocity.

17. **(2)**

$$I = \frac{ML^2}{3}\sin^2 45° \times 2 = \frac{ML^2}{3}$$

18. **(1)**

$\cos\phi = 1 \Rightarrow \phi = 0° \Rightarrow \omega L = 1/\omega C$

19. **(1)**

Density ρ of gas is given by relation PM=ρRT

$$\Rightarrow \frac{P}{T} = \text{Constant} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{2T_1} \Rightarrow P_2 = 2P_1$$

Hence pressure becomes double, which means 100% increase.

20. **(2)**

Net force on $-q_0$ provides the required centripetal force. This gives $v = \frac{x}{\left(a^2 + x^2\right)^{3/4}}$

For maximum speed $\frac{dv}{dx} = 0 \Rightarrow x = \sqrt{2}d$

21. **(1)**

(1kg). 3m/s = (3kg).v (by momentum conservation)

$W = \Delta K.E$

$$W = \frac{1}{2}(2)(1)^2 \text{ Joule} = 1\text{Joule}$$

22. **(15)**

1^{st} case : Image is formed at R_1

$$-\frac{1}{f_{m_1}} = \frac{2}{f_i} + \frac{1}{(-f_m)} = \frac{2}{15}$$

$$\therefore R_1 = 2f_{m_1} = -15\text{cm}$$

2^{nd} case : Image is formed at R_2

$$\frac{-1}{f_{m_2}} = \frac{2}{f_{i_g}} + \frac{2}{f_{w_g}} + \frac{1}{f_m}$$

$$\Rightarrow f_{m_2} = -\frac{90}{13}\text{cm}$$

$$\therefore R_2 = -\frac{180}{13} \text{ cm}$$

Shift in pin, so that its image again coincides is

$\Delta x = x_1 - x_2 = 15 - 180/13 = 1.15$ cm (down)

23. **(5)**

Magnetic field due to wire

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{30}{2 \times 10^{-2}} = 3 \times 10^{-4} T$$

This magnetic field will be perpendicular to external magnetic field

$\therefore$ Net magnetic field

$$B = \sqrt{B^2 + B_0^2}$$

$$= \sqrt{\left(3 \times 10^{-4}\right)^2 + \left(4 \times 10^{-4}\right)^2}$$

$$= 5 \times 10^{-4} T$$

24. **(30)**

$$H_1 = \frac{u^2 \sin^2 45°}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \text{ ....}(i)$$

When half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30°}{2g} = \frac{u^2}{16g} \text{ ....}(ii)$$

Form (i) and (ii)

$$H_2 = \frac{H_1}{4} = 30\text{m}$$

25. **(3)**

The magnetic file due to a straight current – carrying wire is

$$B_{PQR} = 2 \times \frac{\mu_0\left(\frac{2i}{3}\right)}{4\pi\left(\frac{\alpha}{2}\right)}\left[\sin 45° + \sin 45°\right]$$

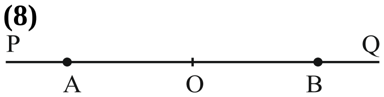
$$B_{PSR} = 2 \times \frac{\mu_0\left(\frac{i}{3}\right)}{4\pi\left(\frac{\alpha}{2}\right)}\left[\sin 45° + \sin 45°\right]$$

$$B_{net} = B_{PQR} - B_{PSR}$$

$$\Rightarrow B_{net} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$$

$$\Rightarrow k = 3$$

26. **(8)**



Let O be the mean position.
As time from A to O = Time from O to B
Hence AO = OB. And hence PA = QB
$\therefore$ Time period = $t_{AB} + t_{BQB} + t_{BA} + t_{APA}$
$= (2+2+2+2)\text{s} = 8\text{s}$

27. **(9)**
In young's double slit experiment, intensity at a point is given by

$I = I_0 \cos^2 \frac{\phi}{2}$ ....(i)

where, ϕ = phase difference,
For path difference λ, phase difference $\phi_1 = 2\pi$
For path difference $\frac{\lambda}{6}$, phase difference $\phi_2 = \frac{\pi}{3}$
Using equation (i),

$$\frac{I_1}{I_2} = \frac{\cos^2\left(\frac{\phi_1}{2}\right)}{\cos^2\left(\frac{\phi_2}{2}\right)} = \frac{\cos^2\left(\frac{2\pi}{2}\right)}{\cos^2\left(\frac{\pi}{3}\right)}$$

$$\Rightarrow \frac{K}{I_2} = \frac{1}{\frac{3}{4}} = \frac{4}{3} \Rightarrow I_2 = \frac{3K}{4} = \frac{9K}{12}$$

28. **(4)**
At$(3,3)$, $U_i = 18\alpha$
At$(1,1)$, $U_f = 2\alpha$
Now, $K_i + U_i = K_f + U_f$
or, $0 + 18\alpha = K + 2\alpha$
or, $K = 16\alpha$

29. **(3)**
Applying $\sum \vec{F} = m\vec{a}$ to m_1 and m_2 gives
$m_2 g - m_1 g \sin\theta = (m_1 + m_2)a$
Putting values of m_1, m_2 and θ gives
$a = \frac{5}{3}\text{m/s}^2$

30. **(2)**
Flux passing from the small loop

$$e = \frac{d\phi}{dt} = \left(\frac{\mu_0 a^2}{2R}\right)\frac{di}{dt} \text{ (in volt)}$$

$$= \left(\frac{\mu_0 a^2}{2R}\right)\left(\frac{i_0}{\tau}\right) \times 10^6 \text{ (in microvolt)}$$

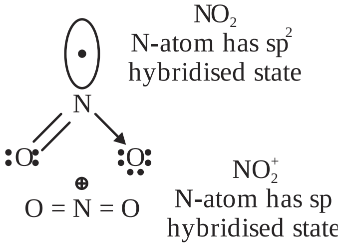
$$= (4\pi \times 10^{-7})(10^{-2})^2 \times \frac{2}{\pi} \times 100 \times 10^6 \times \frac{1}{2 \times 0.1 \times 2 \times 10^{-2}}$$

$= 2\mu\text{V}$

SECTION-II (CHEMISTRY)

31. **(1)**
Given, $2C + 3H_2 \rightarrow C_2H_6$; $\Delta H = -21.1$
$C + O_2 \rightarrow CO_2$; $\Delta H = -94.1$
$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$; $\Delta H = -68.3$
Eqs. 2 × (ii) + 3 × (iii) – (i)
$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$
$\Delta H = 2(-94.1) + 3(-68.3) - (-21.1)$
$= -372$ kca1/mol

32. **(2)**



33. **(2)**
+5 oxidation state of nitrogen does not undergo disproportionation.

34. **(3)**
K.E. = $h\nu - h\nu_0$
Slope of K.E. and ν plot gives value of planck's constant.

35. **(3)**
Combination A, C, E will give molar conductivity of $Ba(OH)_2$ at infinite dilution.

36. **(1)**
This is an example of oxymercutation - demercuration and gives alcohol corresponding to Markovnikov's addition of water to carbon-carbon double bond.

$$CH_3-CH_2-\underset{\substack{|\\CH_3}}{C}=CH_2 \xrightarrow{Hg(CH_3COO)_2H_2O}$$

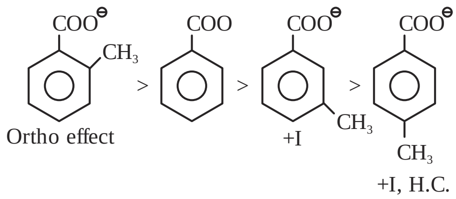
$$CH_3-CH_2-\overset{\substack{CH_3\\|}}{\underset{\substack{|\\OH}}{C}}-\underset{\substack{|\\HgOCOCH_3}}{CH_2}$$

$$\xrightarrow{NaBH_4} CH_3-CH_2-\overset{\substack{CH_3\\|}}{\underset{\substack{|\\OH}}{C}}-CH_3$$

$(2-\text{methylbutan}-2-\text{ol})$

37. **(4)**

Order of stability of conjugate base is



38. **(2)**

$PCl_5 \rightleftharpoons PCl_3 + Cl_2$

1 0 0

$1-\alpha$ α α

$$\therefore \quad K_P = \frac{\left(\frac{\alpha P}{1+\alpha}\right)\left(\frac{\alpha}{1+P}P\right)}{\left(\frac{1-\alpha}{1-\alpha}\right)p}$$

$$= \frac{\alpha^2}{1-\alpha^2}P \approx \alpha^2 P(\alpha << 1)$$

Neglecting α^2

$K_P = \alpha^2 P$

$$\therefore \quad \alpha = \frac{\sqrt{K_P}}{\sqrt{P}}$$

39. **(3)**

$[A]_t = [A] - kt = 1 - 0.001 \times 10 \times 60$

$= 0.4$ M

$[B]_t = 0.001 \times 10 \times 60 = 0.6M$

40. **(4)**

Hydrolysis reactions of $SbCl_3$ and $BiCl_3$ are reversible and partial, forming white ppts of SbOCl and BiOCl. These dissolve on adding dil HCI due to reverse reaction

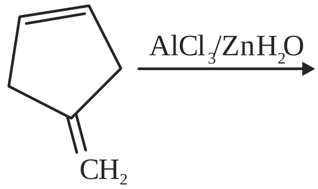
$SbOC1_3 + H_2O \rightleftharpoons SbOCl + 2HC1$

$BiC1_3 + H_2O \rightleftharpoons BiOCl + 2HC1$

41. **(2)**

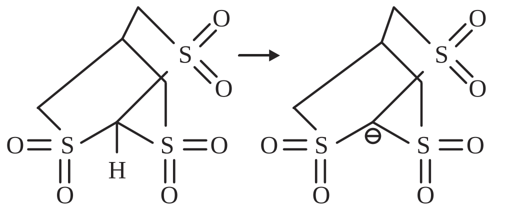
Number of solute particles are same in (iii).

42. **(2)**

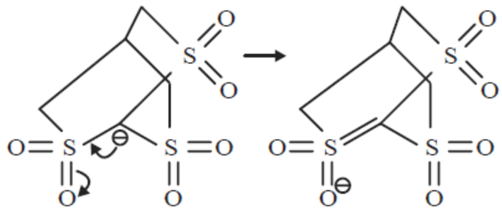


$$H-\overset{\substack{O\\||}}{C}-CH_2-\overset{\substack{O\\||}}{C}-CH_2-\overset{\substack{O\\||}}{C}-H + HCHO$$

43. **(1)**



Only-1 effect



Sulphar has vacant d-obitals which are more diffused hence can formπ-bond on bridge

44. **(4)**

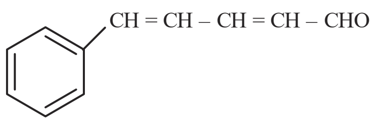
In HF - - - - $F^{\ominus}$, H–bond is as strong as a covalent bond.

45. **(3)**

For

$$CO_3^{-2}\ B.O = 1 + \frac{\text{No. of } \pi\text{-bonds}}{\text{Total no. of } \sigma \text{ bonds}} = 1 + \frac{1}{3} = 1.33$$

46. **(1)**



47. **(2)**

Fe^{2+} has configuration

|A| $4s^0 3d^6$

If complex is diamagnetic then ligand must be strong field while in paramagnetic ligand must be weak.

48. (1)

$NaBH_4$ reduces

$\gt C=O$ as well as $-\overset{\overset{O}{||}}{C}-Cl$ group.

49. (1)

Methyl pyranoside contain stable acetal linkage, hence no mutarotation.

50. (3)

$VC1_4$ is the molecular formula has one unpaired electron (magnetic moment = 1.73) which means it is present in +4 Oxidation state ($3d^1$)

51. (4)

$A^0\ m(AgBr) = \lambda^0 Ag^{\oplus} + \lambda^0 Br^{\oplus}$

$= 62 + 78 = 140.0\ S\ cm^2\ mol^{-1}$

$\therefore\quad A^0\ m(AgBr) = A_{m(AgBr)} = \dfrac{K \times 1000}{S}$

$140.0\ S\ cm^2\ mol{-}1 = \dfrac{8.5\times10^{-7}\times10^3}{S}$

$\therefore\quad S = 8.5 \times 10^{-7} \times 10^3/140.0$

$= 0.06 \times 10^{-4}\ Mol\ L^{-1}$

$= 0.06 \times 10^{-4}\ Mw_{(AgBr)}$

$= 0.06 \times 10^{-4} \times 188\ g\ L^{-1}$

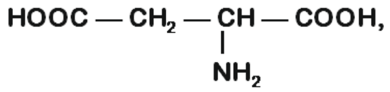
$= 1.128 \times 10^{-3}\ g\ L^{-1}$

$\therefore\quad K_{SP}$ of $AgBr = S^2M^2 = (0.06 \times 10^{-4})^2$

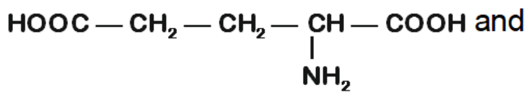
$\simeq 4 \times 10^{-11}$

52. (5)

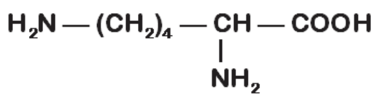
The amino acids present in the given tripeptide Asp – Glu – Lys are



Aspartic acid (Asp)



Glutamic acid (Glu)

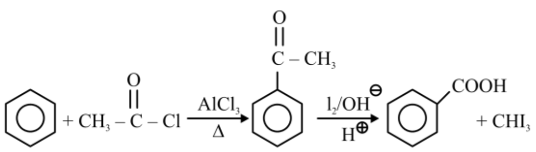


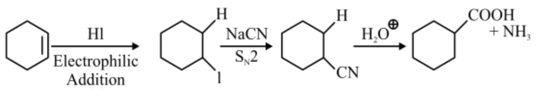
Lysine (Lys)

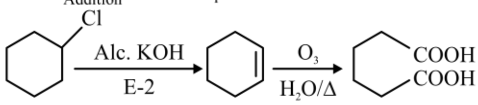
$\therefore$ Number of carbonyl groups present in tripeptide = 2 + 2 + 1 = 5

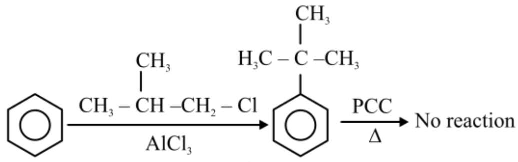
53. (5)

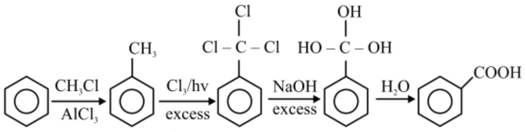
Friedel craft acylation Iodoform reaction

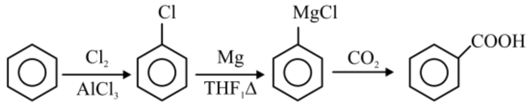
(I) 

(II) 

(III) 

(IV) 

(V) 

(VI) 

54. (87)

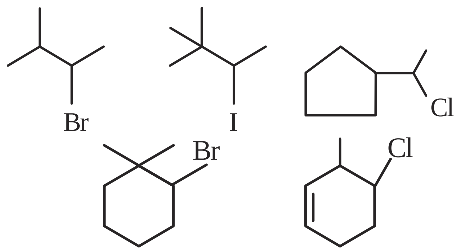
$\alpha = \dfrac{i-1}{n-1} \Rightarrow \dfrac{2.74-1}{3-1}, \alpha = 0.87, \therefore \%$ of $\alpha = 87\%$

55. (400)

$\Delta S = \dfrac{\Delta H(vap)}{(B,P)} = \dfrac{30KJ}{75} = 400K$

56. (5)

Five Halides will undergo rearrangement in S_N1 reaction.



57. (285)

CFSE for the d^6 weak field is $-0.4\Delta_0$. The ΔH_{hyd} is higher because the energy of the complex is lowered by that much

CFSE = –11.4 kca1/mol

–11.4 kcal / mol = $0.4\Delta_0$

$\Delta_0 = \dfrac{11.4}{0.4} = 28.5\ k.cal / mol^{-1}$

58. (5)

Eq of $Fe_2(C_2O_4)_3$ = eq. of $K_2Cr_2O_7$

5 × 6 = mol × 6

Mol of $K_2Cr_2O_7$ = 5

59. (5)

Except statement 4 all are correct, its double bond equivalence is 5

60. (4)

Co is sp^3d^2 hybridised with 4 unpaired electrons in the complex

SECTION-III (MATHEMATICS)

61. (2)

$$\sum \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1+2^r \cdot 2^{r-1}}\right)$$

$$= \sum_{r=1}^{n}[\tan^{-1}(2^r) - \tan^{-1}(2^{r-1})]$$

$$= \tan^{-1}(2^n) - \tan^{-1}(1)$$

$$= \tan^{-1}(2^n) - \frac{\pi}{4}$$

62. (3)

Let $f(1) = x$

Then $f(1 + 1) = x^2$

$f(2 + 1) = x^3$

and so on.

$$\sum_{r=1}^{\infty} f(r) = 31 \Rightarrow x + x^2 + x^3 + ...\infty = 31$$

$$\Rightarrow \frac{x}{1-x} = 31$$

$\Rightarrow \quad x = 31 - 31x$

$32x = 31$

$$x = \frac{31}{32}$$

$$\frac{f(9)+f(10)}{f(9)+f(10)} = \frac{x^9 + x^{10}}{x^9 - x^{10}} = \frac{1+x}{1-x}$$

$= 63$

63. (1)

$$\mathrm{l} = \int_{-\infty}^{0} e^x dx + \int_{0}^{\ln 2} (e^x - 1)dx + \int_{\ln 2}^{\ln 3} (e^x - 2)dx$$

On solving $= 3 + \ln 2 - 2 \ln 3$

64. (3)

$4x^2 + 2x - 1 = 0$

$$\beta + \alpha = -\frac{1}{2} = \alpha = \frac{-1}{2} - \beta$$

$$\alpha = -\frac{1}{2}[4\beta^2 + 2\beta] - \beta$$

$= -2\beta^2 - \beta - \beta = -2\beta^2 - 2\beta$

$= -2\beta^2 + 4\beta^2 - 1 = 2\beta^2 - 1$

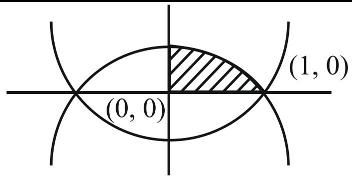
65. (2)

$y^2 = (x^2 - 1)^2$

So $y = x^2 - 1$ or $y = 1 - x^2$

So required area

$$= 4\int_0^1 (1 - x^2)dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = \frac{8}{3}$$



66. (3)

Let the number be $(a - d)$, a, $(a + d)$

$3a = 24 \Rightarrow a = 8$

Also $(8 - d)(8)(8 + d) = 384$

$d^2 = 16$

$d = 4$ or -4

Hence series is 4, 8, 12, …

$\Rightarrow S_n = 2n(n + 1)$ or 12, 8, 4, …

$\Rightarrow S_n = 14n - 2n^2$

67. (2)

Let w be the event that the ball drawn is white

Then $P(w) = \dfrac{x}{y}$

When, 1 ball is lost

$$P(w) = \frac{x}{y}\frac{x-1}{y-1} + \frac{y-x}{y}\frac{x}{y-1} = \frac{x}{y}$$

When 2 ball is lost

$$P(w) = \frac{{}^xC_2}{{}^yC_2}\frac{x-2}{y-2} + \frac{{}^xC_1\,{}^{y-x}C_1}{{}^yC_2}\frac{x-1}{y-2} +$$

$$\frac{{}^{y-x}C_2}{{}^yC_2}\frac{x}{y-2} = \frac{x}{y}$$

Similarly, for $n = 3$ $P(w) = \dfrac{x}{y}$

68. (1)

$x^2 + y^2 - 2x - 6y + 6 = 0$ centre $(1, 3)$

$$r_1 = \sqrt{1+9-6} = 2$$

$$CM = \sqrt{1+4} = \sqrt{5}$$

$$r = \sqrt{5+4} = 3$$

69. (3)

$x^2 + 2x + 1 + 4[y^2 + 2y + 1] - 5 - P = 0$

$$\Rightarrow \frac{(x+1)^2}{P+5} + \frac{(y+1)^2}{\frac{P+5}{4}} = 1$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2\left(\frac{P+5}{4}\right)}{\sqrt{5+P}} = 4$$

$$\Rightarrow \sqrt{P+5} = 8 \Rightarrow P = 59$$

Major axis $2a = \sqrt{P+5} = 2 \times 8 = 16$

70. (3)

$\cos\alpha = \frac{1}{2}\left(x+\frac{1}{x}\right)$ and $\cos\beta = \frac{1}{2}\left(y+\frac{1}{y}\right)$

Since $xy > 0 \Rightarrow x+\frac{1}{x} \ge 2$ or ≤ -2

$y+\frac{1}{y} \ge 2$ or $\le$ -2

$\Rightarrow \cos\alpha = 1, \cos\beta = 1$ …(1)

or

$\Rightarrow \cos\alpha = -1, \cos\beta = -1$ …(1)

$\therefore \cos\alpha\cos\beta = 1$

$(\cos\alpha + \cos\beta)^2 = 4$

$\Rightarrow \alpha + \beta$ is an even multiple of π

$\Rightarrow \sin(\alpha + \beta + \gamma) = \sin(2n\pi + \gamma) = \sin\gamma$

Also, $\sin\alpha = \sin\beta = 0$

71. (4)

Given lines in the form $a(1) + b(-2) + c = 0$

Being lines concurrent, triangle will not form.

72. (1)

$\frac{ydy}{1-y^2} = \frac{dx}{x}$

Integrating we get, $-\frac{1}{2}\ell n|1-y^2| = \ell n(cx)$

$\Rightarrow (1-y^2)x^2 = c$ But $f(4, 0) = 0 \Rightarrow c = 16$

$\therefore$ Solution curve is $(1-y^2)x^2 = 16$

Which is symmetric w.r.t. x-axis.

73. (1)

$(xf'(x))^4 - \int_1^4 f'(x)dx$

$4f'(4) - f'(1) - (f(4) - f(1)) = 6$

74. (1)

$\arg\left(\frac{z-2i}{z+2i}\right) = \frac{\pi}{6} \Rightarrow \arg(z-2i) - \arg(z+2i) = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left(\frac{y-2}{x}\right) - \tan^{-1}\left(\frac{y+2}{x}\right) = \frac{\pi}{6}$

$\Rightarrow \frac{xy-2x-xy-2x}{x^2+y^2-4} = \frac{1}{\sqrt{3}}$

$\Rightarrow x^2+y^2-4 = -4\sqrt{3}x$

$\Rightarrow x^2+y^2+4\sqrt{3}x-4 = 0$

$\Rightarrow (x+2\sqrt{3})^2 + y^2 = 12+4 = 4^2$

$\therefore$ centre $(-2\sqrt{3}, 0)$, radius = 4

(Major arc of circle)

75. (3)

Shortest distance = $\dfrac{\left\|\begin{matrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2k & 3 & -12 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{matrix}\right\|}$

$\Rightarrow 13 = \frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$

$\Rightarrow 13 = \frac{|153+8k|}{\left|4^2+3^2+(-12)^2\right|}$

$\Rightarrow 13 = \frac{|153+8k|}{13}$

$|153 + 8k| = 169$

$153 + 8k = 169, -169$

$k = \frac{16}{8}, \frac{-322}{8}$

$\Rightarrow k = 2, -40.25$

$4\left|\sum_{k\in S} k\right| = 153$

76. (2)

$f(x) = 2 + \frac{1}{\left(\cos x + \frac{1}{2}\right)^2 + \frac{3}{4}}$

$f(x)_{\min} = \frac{7}{3}, f(x)_{\max} = \frac{10}{3}$

77. (1)

Given that $|z| = 1$ and $\frac{z-1}{z+1}(z \ne -1)$

Now, we know that $z\bar{z} = |z|^2$

$\Rightarrow z\bar{z} = 1(\text{for } |z| = 1)$

$\Rightarrow \left[\frac{z-1}{z+1}\right] \times \frac{(\bar{z}+1)}{(\bar{z}+1)} = \frac{z\bar{z}+z-\bar{z}-1}{z\bar{z}+z+\bar{z}+1} = \frac{2iy}{2+2x}$

($\because z\bar{z} = 1$ and taking $z = x+iy$ so that $z+\bar{z} = 2x$ and $z-\bar{z} = 2iy$)

$\Rightarrow \frac{z-1}{z+1} = 0 + i\left(\frac{\text{Im}(z)}{1+\text{Re}(z)}\right)$

78. (3)

$\because \left(x-\sqrt{x^2-1}\right)^6 + \left(x+\sqrt{x^2-1}\right)^6$

$= 2\left[{}^6C_0x^6 + {}^6C_2x^4\left(x^2-1\right) + {}^6C_4x^2\left(x^2-1\right)^2 + {}^6C_6\left(x^2-1\right)^3\right]$

$= 2[32x^6 - 48x^4 + 18x^2 - 1]$

$\therefore\quad a$ = coefficient of $x^4 = -96$

b = coefficient of $x^2 = 36$

$\Rightarrow\quad a - b = -96 - 36 = -132$

$a + b = -96 + 36 = -60$

79. **(2)**

Let $z = x + iy$

$\therefore\quad 2x + 2iy = \sqrt{x^2 + y^2} + 2i$

$\therefore\quad 2x - \sqrt{x^2 + y^2} = 0,\ 2y - 2 = 0$

$\therefore\quad y = 1$ and $x = \dfrac{1}{\sqrt{3}}$

$\therefore\quad z = \dfrac{1}{\sqrt{3}} + i$

80. **(4)**

We have

$$T = \int_0^{\ln 2} \frac{(3e^{3x} + 2e^{2x} - e^x) - (e^{3x} + e^{2x} - e^x + 1)}{e^{3x} + e^{2x} - e^x + 1}\,dx$$

$$= [\ln(e^{3x} + e^{2x} - e^x + 1) - x]_0^{\ln 2}$$

$$= \ln\frac{11}{2} - \ln 2 = \ln\frac{11}{4} \Rightarrow e^T = e^{\ln\frac{11}{4}} = \frac{11}{4}$$

81. **(2)**

$\dfrac{dy}{dx} - \dfrac{y}{x} = -x \qquad \left(I.F. = \dfrac{1}{x}, y(0) = 1\right)$

$\Rightarrow\quad y \cdot \dfrac{1}{x} = \int -dx \Rightarrow \dfrac{y}{x} = -x + c$

$\Rightarrow\quad y = x - x^2$

$$A = \int_0^1 \left(x - x^2\right) dx = \frac{1}{6}$$

82. **(16)**

$$\lim_{x\to 0} B\left(\frac{e^x + e^{-x} - 2\cos x}{3x^2}\right)$$

$$= \lim_{x\to 0} \frac{B(e^x - e^{-x} + 2\sin x)}{6x}$$

$2B = 3A$

sample space $= {}^{10}C_2 = 45$

favourable (A, B) are (2, 3), (4, 6), (6, 9)

83. **(2)**

Two plane figures are triangle and trapezium

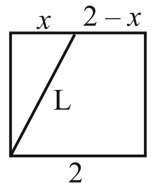
Let side of triangle be x and its permitter be P_1

$\Rightarrow P_1 = x + 2 + L$, where L is length of cut.

Now, perimeter of trapezium $= P_2$

$= 2 + 2 + (2 - x) + L = 6 - x + L$

$\Rightarrow$ sum of the perimeters $= 8 + 2L$



Which is maximum, when L is maximum

$\Rightarrow L^2 = 4 + 4 \Rightarrow L = 2\sqrt{2}$

$\Rightarrow P = 8 + 4\sqrt{2}$

84. **(1)**

Let $A = (x - 1)(x^2 - 2) \ldots (x^{20} - 20)$

$$= x \cdot x^2 \cdot x^3 \ldots x^{20}\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x^2}\right)\ldots\left(1 - \frac{20}{x^{20}}\right)$$

$$\therefore\ A = x^{210}\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x^2}\right)\ldots\left(1 - \frac{20}{x^{20}}\right)$$

$\therefore$ Coefficient of x^{210} in A is 1.

85. **(63)**

$1 + 49 + 49^2 + \ldots.. + (49)^{125}$

$$\frac{(49)^{126} - 1}{48} = \frac{\left((49)^{63} - 1\right)\left((49)^{63} + 1\right)}{48}$$

Greatest value of p is 63.

86. **(4)**

$$K = {}^9C_3 \times {}^3C_2 \times \frac{5!}{2!\ 2!\ 1!}$$

87. **(5)**

Area of $\Delta ABC = \dfrac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

Now given $2\vec{a} + 3\vec{b} + 6\vec{c} = 0$

Cross with $\vec{a}, 3\vec{a} \times \vec{b} + 6\vec{a} \times \vec{c} = 0$

or $\vec{a} \times \vec{b} = 2(\vec{c} \times \vec{a})$

Again, cross with $\vec{b}$

$2\vec{a} \times \vec{b} + 6\vec{c} \times \vec{b} = 0$

or $\vec{a} \times \vec{b} = 3(\vec{b} \times \vec{c})$

Area of $\Delta AOB = \dfrac{1}{2}|\vec{a} \times \vec{b}|$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta AOB} = \frac{\frac{1}{2}|\vec{a} \times \vec{b}|\left(1 + \frac{1}{2} + \frac{1}{3}\right)}{\frac{1}{2}|\vec{a} \times \vec{b}|} = \frac{11}{6}$$

So, $m = 11, n = 6 \Rightarrow (m - n) = 5$

88. **(4)**

We have, $\displaystyle\int \frac{(\cos x - \sin x)\ dx}{\sqrt{8 - \sin 2x}}$

$$= \int \frac{(\cos x - \sin x)\,dx}{\sqrt{9-(\sin x + \cos x)^2}}$$

Put $(\sin x + \cos x) = t \Rightarrow (\cos x - \sin x)\,dx = dt$

$$= \int \frac{dt}{\sqrt{3^2 - t^2}} = \sin^{-1}\frac{t}{3} + c$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

$\Rightarrow \quad \alpha = 1, \beta = 3 \therefore (\alpha, \beta) \equiv (1, 3)$

89. **(6)**

$B^T B^{-1} = A \Rightarrow |A| = |B^T|\,|B^{-1}| = |A| = 1$

take adjoint both sides

adj (B^{-1}) . adj (B^T) = adj A (adj $A = B$)

$(\text{adj } B)^{-1} (\text{adj } B)^T = B$

$(\text{adj } B)^T = (\text{adj } B)(B)$

$\Rightarrow (\text{adj } B)^T = |B|\, I$

Adj $B = |B|\, I$

$|B| \cdot B^{-1} = |B|\, I$

$B = I$

$A = I$

90. **(2)**

Let $x = \frac{1}{t}$

$$\lim_{t\to 0}\left(\frac{f(3+3t)}{f(3)}\right)^{\frac{1}{t}} = e^{\lim_{t\to 0}\left(\frac{f(3+3t)-f(3)}{tf(3)}\right)}$$

$$= e^{\lim_{t\to 0}\left(\frac{f'(3+3t).3}{f(3)}\right)} = e$$