All India Test Series (JEE–2024)

AVJLM1/05

Test- 05

Lakshya JEE 2024

DURATION : 180 Minutes | DATE : 12/01/2024 | M. MARKS : 300

ANSWER KEY

PHYSICS

1. (2)
2. (2)
3. (3)
4. (1)
5. (4)
6. (3)
7. (4)
8. (2)
9. (4)
10. (1)
11. (1)
12. (2)
13. (3)
14. (3)
15. (3)
16. (1)
17. (2)
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19. (1)
20. (1)
21. (7)
22. (50)
23. (2)
24. (25)
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26. (1)
27. (397)
28. (6)
29. (50)
30. (5)

CHEMISTRY

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59. (10)
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MATHEMATICS

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81. (12)
82. (54)
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86. (2)
87. (3)
88. (2)
89. (32)
90. (18)

SECTION-I (PHYSICS)

1. **(2)**

For the two blocks to move together

$$\mu_{\min} = \frac{F}{(M+m)g} = \frac{F}{2Mg}$$

As $\mu > \mu_{\min}$,

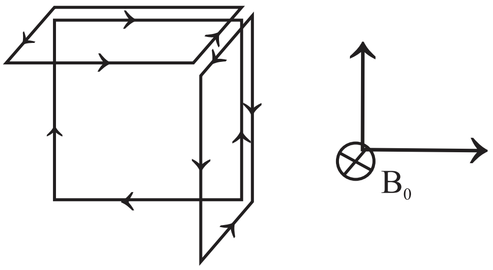
The two blocks move together.

$\therefore$ Velocity of the system (M + m) is

$$v = at = \left(\frac{F}{2M}\right)t$$

$$P = Fv = F\frac{F}{2m}t = \frac{F^2}{2M}t$$

2. **(2)**



$$B = \sqrt{B_0{}^2 + B_0{}^2 + B_0{}^2} = \sqrt{3}B_0 \text{ towards } F$$

3. **(3)**

$$v = \frac{50}{100}V_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

Applying energy conservation

$$\Rightarrow -\frac{GmM}{R} + \frac{1}{2}mv^2 = -\frac{GmM}{(R+h)}$$

$$v^2 = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow \frac{1}{4R} = \frac{h}{R(R+h)} \Rightarrow R+h = 4h \Rightarrow h = R/3$$

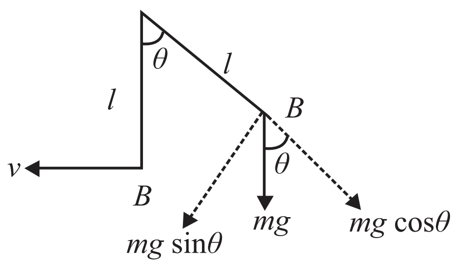
4. **(1)**

$$a_A = g\sin\theta \text{ (only tangential)}$$

$$a_B = \frac{v^2}{l} \text{ (only radial)}$$

$$K.E. + P.E. = K.E. + P.E.$$

$$= \frac{1}{2}m0^2 + mgl(1-\cos\theta) = \frac{1}{2}mv^2$$



$$v^2 = 2gl(1-\cos\theta) \qquad ....(i)$$

$$\therefore\ a_B = \frac{v^2}{l} = 2g(1-\cos\theta)$$

$$\Rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{1}{2}$$

5. **(4)**

Centre of mass of circular disc of radius 4R = (0, 0)

Centre of mass of upper disc = (0,3R)

Centre of mass of lower disc = (3R, 0)

Let M be mass of complete disc and then the mass of each cut out disc is $\frac{M}{16}$

Hence, centre of mass of new structure is given by

$$\vec{x} = \frac{m_1x_1 - m_2x_2 - m_3x_3}{m_1 - m_2 - m_3}$$

$$x = \frac{M(0) - \frac{M}{16}(0) - \frac{M}{16}(3R)}{M - \frac{M}{16} - \frac{M}{16}} = \frac{-3R}{14}$$

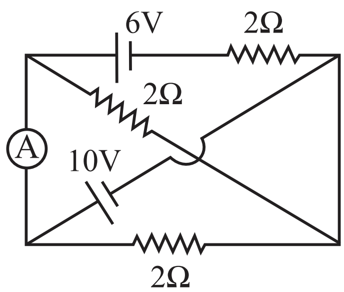
$$y = \frac{m_1y_1 - m_2y_2 - m_3y_3}{m_1 - m_2 - m_3}$$

$$\Rightarrow \vec{y} = \frac{M(0) - \frac{M}{16}(0) - \frac{M}{16}(3R)}{M - \frac{M}{16} - \frac{M}{16}} = \frac{-3R}{14}$$

Position vector of $C.M. = -\frac{3R}{14}(\hat{i} + \hat{j})$

6. **(3)**

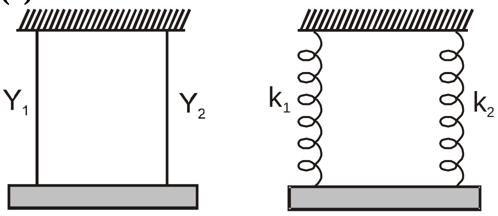
Applying Kirchhoff's law in different loops gives current through ammeter = (2+5) = 7A



7. **(4)**

Magnetic flux through the loop is constant. Hence no emf is induced and hence no induced current in the loop.

8. **(2)**



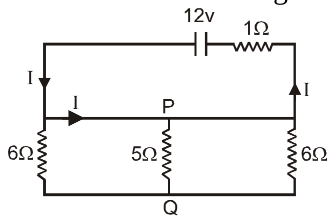
$K_{eq} = K_1 + K_2$

$$\frac{Y2A}{\ell} = \frac{Y_1A}{\ell} + \frac{Y_2A}{\ell}$$

$$Y = \frac{Y_1+Y_2}{2}$$

9. **(4)**

Just after switch closing



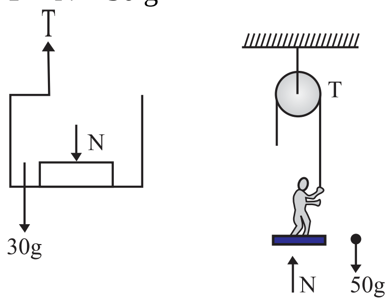
current through resistor PQ is zero just after closing the switch.

10. **(1)**

Applying Newton's second law,

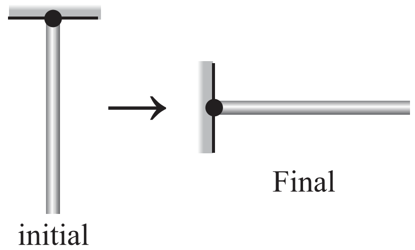
T = N+30 g

T + N = 50 g



Solving N = 100 Newton.

11. **(1)**



Total mechanical energy initially $= -Mg\dfrac{L}{2}$

(gravitational P.E.)

+0 (Electrical P.E.) + 0 (Kinetic Energry)

Total mechanical energy finally = 0 (Gravitational P.E.)

$-(Q.E.)\left(\dfrac{L}{2}\right)$(Electrical P.E.) + 0 (Kinetic Energy)

$$\Rightarrow Mg\frac{L}{2} = Q.E.\left(\frac{L}{2}\right) \qquad \Rightarrow E = \frac{Mg}{Q}$$

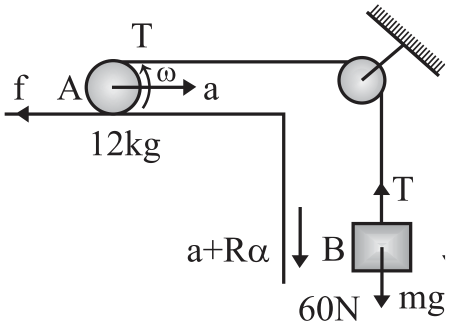
12. **(2)**

Equation of linear motion

Block 60-T = 6 (a+$R\alpha$) …………..(1)

Cyl T-f = 12.a …………..(2)

Rotation $a = R\alpha$ …………..(3)



Also $(T+f)R = \dfrac{MR^2}{2}\alpha$

$\Rightarrow T + f = \dfrac{MR\alpha}{2} = 6R\alpha$ …………(4)

60 - T = 12 $R\alpha$ …………(5)

$T - f = 12R\alpha$ …………(6)

Multiply (5) by 2 & add all (4), (5) & (6)

120 = 42 $R\alpha$ = 42a

$$a = \frac{120}{42} = \frac{2g}{7}$$

(Ans : 2g/7)

13. **(3)**

$m_1v_1 = m_2v_2 = P$

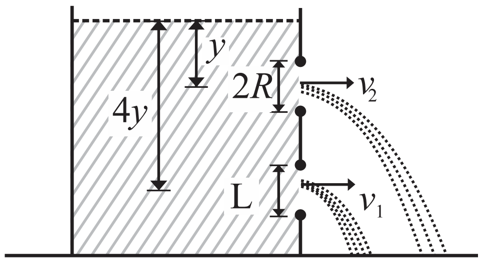
K.E. $= \dfrac{1}{2}mv_1^2 + \dfrac{1}{2}m_2v_2^2$

$$= \frac{1}{2}m_1\left(\frac{P}{m_1}\right)^2 + \frac{1}{2}m_2\left(\frac{P}{m_2}\right)^2 = \frac{1}{2}$$

$$\frac{P^2(m_2+m_1)}{m_1m_2}$$

14. **(3)**

Let v_1 and v_2 be the velocity of efflux from square and circular holes respectively. S_1 and S_2 be cross-section areas of square and circular holes.



$$v_1 = \sqrt{2g(4y)}$$

and $v_2 = \sqrt{2g(y)}$

The volume of water coming out of square and circular hole per second is

$Q_1 = v_1S_1 = \sqrt{8gy}L^2 \quad ; \quad Q_2 = v_2S_2 = \sqrt{2gy}\,\pi R^2$

$\because Q_1 = Q_2, \qquad R = \sqrt{\frac{2}{\pi}}.L$

15. **(3)**

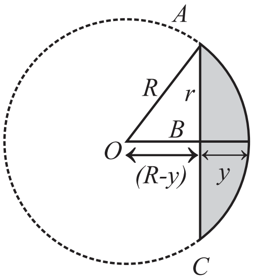
According to lens formula $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

The lens is plano-convex i.e., $R_1 = R$ and $R_2 = \infty$

Hence $\frac{1}{f} = \frac{\mu - 1}{R} \Rightarrow f = \frac{R}{\mu - 1}$

Speed of light in medium of lens $v = 2\text{x}10^8$ m/s

$\Rightarrow \mu = \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = \frac{3}{2} = 1.5$



If r is the radius and y is the thickness of lens (at the centre), the radius of curvature R of its curved surface in accordance with the figure is given by

$R^2 = r^2 + (R - y)^2 \Rightarrow r^2 + y^2 - 2Ry = 0$

Negalecting y^2 ; we get $R = \frac{r^2}{2y} = \frac{(6/2)^2}{2\times0.3} = 15\text{cm}$

Hence $f = \frac{15}{1.5 - 1} = 30\text{cm}$

16. **(1)**

From 1st maxima to 5th minima, number of fringes =3.5

$\Rightarrow \beta = \frac{7\text{mm}}{3.5} = 2\text{mm}$

from $\beta = \frac{\lambda D}{d}$, $\lambda = \frac{BD}{d} = 600\,\text{nm}$

17. **(2)**

For α decay, number of protons decreases by 2.
For β^- decay, number of protons increases by 1.
In β^+ decay, number of protons decreases by 1.
In K capture, a proton changes into a neutron.

18. **(3)**

Energy is released in a process when total Binding energy (B.E.) of the nucleus is increased or we can say when total B.E. of products is more than the reactants.. By calculation we can see that only in case of option (3), this happens.
Given W→ 2Y
B.E. of reactants = 120 × 7.5 = 900 MeV and B.E. of products = 2 × (60 × 8.5) = 1020 MeV
i.e. B.E. of products > B.E. of reactants.

19. **(1)**

Redrawing the circuit gives equivalent capacitance.

20. **(1)**

$e = \frac{v\sin\theta}{\sqrt{2gh}\cos\theta}$

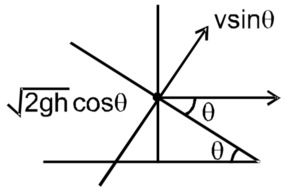
apply conservation of momentum

$m\sqrt{2gh}\sin\theta = m\,v\cos\theta \qquad$ ......(i)

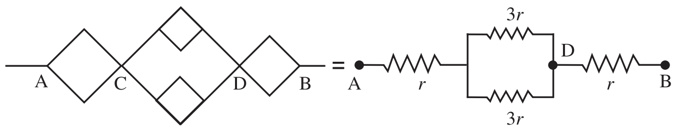
$e\sqrt{2gh}\cos\theta \times m = mv\cos\theta \qquad$ ......(ii)

$\frac{\tan\theta}{e} = \cot\theta$

$\therefore \quad e = \tan^2\theta$



21. **(7)**

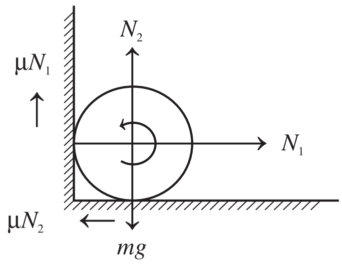


The circuit can be simplified as shown in the above figure. Due to symmetry, we can see that the distribution of current at point A in the circuit, is the same as the recombination of the current before point B. Also, the junction which is present between the points C and D can be disconnected because it won't affect the distribution of current.

$r_{AB} = r + 1.5r + r$

$\Rightarrow r_{AB} = 3.5r = 7\Omega$

22. **(50)**



$\mu N_1 + N_2 = mg \quad$ ....(1)

$N_1 = \mu N_2 \quad$ ....(2)

$N_2 = \frac{mg}{(1+\mu^2)}$

$N_1 = \frac{\mu mg}{(1+\mu^2)}$

The torque about the center of mass of the cylinder is

$$\tau = -(\mu N_1 + \mu N2)R$$

$$\Rightarrow \tau = -\frac{\mu mgR}{1+\mu^2}(\mu+1)$$

From work-energy theorem

$$-\frac{\mu mg(\mu+1)}{(1+\mu^2)} \times \theta = 0 - \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2$$

$$\Rightarrow \theta = \frac{(1+\mu^2)R\omega^2}{4\mu g(\mu+1)}$$

$$\Rightarrow n = \frac{\theta}{2\pi} = \frac{(1+\mu^2)R\omega^2}{8\pi\mu g(\mu+1)}$$

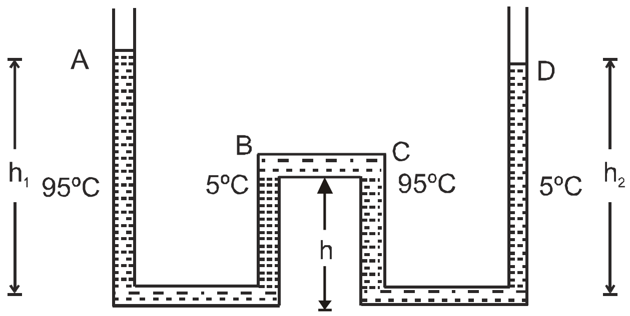
$$\Rightarrow n = 50$$

23. **(2)**

Density of a liquid varies with temperature as–

$$\rho_{t^\circ c} = \left(\frac{\rho_{0^\circ c}}{1+\gamma \;\; t}\right)$$

Here γ is the coefficient of volume expansion of liquid.



In the figure –

h_1 = 52.8 cm, h_2 = 51 cm and h = 49 cm

Now pressure at B = pressure at C.

Therefore,–

$$P_0 + h_1\rho_{95^\circ}\; g - h\rho_{5^\circ}\;\; g = P_0 + h_2\rho_{5^\circ}\; g - h\rho_{95^\circ}\; g$$

$$\Rightarrow \quad \rho_{95^\circ}\; (h_1 + h) = \rho_{5^\circ}\; (h_2 + h)$$

$$\Rightarrow \quad \frac{\rho_{95^\circ}}{\rho_{5^\circ}} = \frac{h_2 + h}{h_1 + h}$$

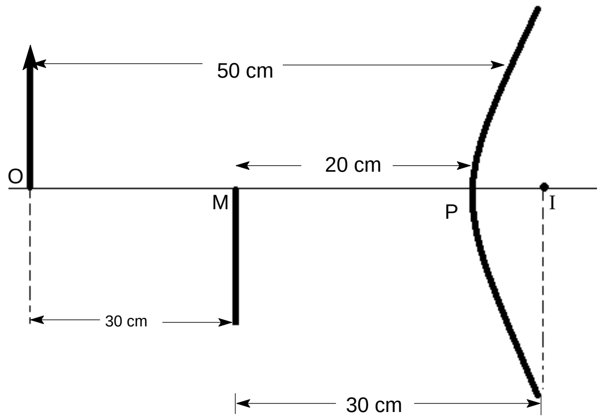
$$\Rightarrow \quad \frac{\dfrac{\rho_{0^\circ}}{(1+95\gamma)}}{\dfrac{\rho_{0^\circ}}{(1+5\gamma)}} = \frac{h_2+h}{h_1+h}$$

$$\Rightarrow \quad \frac{1+5\gamma}{1+95\gamma} = \frac{51+49}{52.8+49} = \frac{100}{101.8}$$

Solving this equation, we get

$\gamma = 2 \times 10^{-4}$ per ºC

24. **(25)**



The plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it.

$\therefore$ The distance of image formed by plane mirror from convex mirror will be

$$PI = MI - MP$$

$$\Rightarrow PI = MO - MP$$

$$= 30 - 20$$

$$= 10\,cm$$

Now as this image coinsides with the image formed by convex mirror, therefore for convex mirror

$u = -50\,cm, v = +10\,cm$

So $\dfrac{1}{10} + \dfrac{1}{-50} = \dfrac{1}{f} \quad \Rightarrow f = 12.5\,cm$

$$R = 2f = 25\,cm$$

25. **(11)**

$$dQ = du + dw$$

$$n.c.dT = nC_V dT + P.dv$$

$$n = 1$$

$$c.dT = \frac{5R}{2}dT + \rho.dv$$

Now, $\rho = \dfrac{K}{T^2}$

$$PV = RT$$

$$V = \frac{RT^3}{K}$$

$$\therefore \quad dV = \frac{3RT^2}{K}dT$$

$$\therefore \quad \int_T^{T+dT} \rho.dV = \int_T^{T+dT} \frac{K}{T^2} \times \frac{3RT^2}{K} \times dT = 3R.dT$$

$$\therefore \quad c = \frac{5R}{2} + 3R = \frac{11R}{2}$$

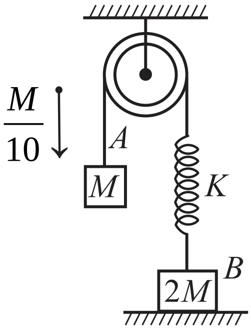
26. **(1)**

From CLM,

$\dfrac{M}{10}v_0 = \dfrac{11}{10}Mv \Rightarrow v = \dfrac{v_0}{11}$ ....(i).

For block B to leave the ground,

$$K\left(x_0+x\right)=2\text{Mg}\left(\text{where, } x_0=\frac{\text{Mg}}{\text{K}}\right)$$



$$\therefore x=\frac{\text{Mg}}{\text{K}} \quad ....(ii).$$

From COE

$$\frac{11}{10}\text{Mg}x-\left[\frac{1}{2}\text{K}\left(x_0+x\right)^2-\frac{1}{2}\text{K}x_2^0\right]$$

$$=0-\frac{1}{2}.\frac{11}{10}\text{M}v^2$$

On solving, $v_0=g\sqrt{\frac{88\text{M}}{\text{K}}}$

On putting the value, we get

$v_0=1\,\text{ms}^{-1}$

27. (397)

$$i=\frac{V}{\text{R}_{eff}}=\frac{V}{1970+30}=\frac{2}{2000}=1\times10^{-3}A=1\text{mA}$$

This current provides full scale deflection (i.e., 20 division) in order to limit the deflection 10 divisions, The resistance needed to connect can be obtained as

$$\theta=\frac{MiAB}{k} \qquad \therefore \frac{\theta_1}{\theta_2}=\frac{i_1}{i_2}$$

$\therefore \theta_1:\theta_2=2$

$\therefore i_1:i_2=2:1$

$$\therefore i_2=\frac{i_1}{2}=\frac{1\times10^{-3}}{2}=5\times10^{-4}\text{mA}$$

$$\therefore i=\frac{v}{\text{R}_{eff}+R_s}$$

$$\Rightarrow 5\times10^{-4}=\frac{2}{30+R_s}$$

$\Rightarrow R_s=3970\,\Omega$

28. (6)

$$\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{1}{970.6\times10^{-10}}=1.097\times10^7\left[\frac{1}{1^2}-\frac{1}{n_2^2}\right]$$

$\Rightarrow n_2=4$

$\therefore$ Number of emission lines

$$N=\frac{n(n-1)}{2}=\frac{4\times3}{2}=6$$

29. (50)

$T'=50\text{ms}$

$$T=2\pi\sqrt{\frac{I}{MB_H}}$$

$$I=\frac{ml^2}{12}\left(m\rightarrow \text{mass of magnet}\right)$$

$$I'=\frac{1}{12}\left[\frac{m}{3}\right]\left[\frac{l}{3}\right]^2\times3=\frac{I}{9}$$

$$M'=M\times\frac{1}{3}\times3=M$$

$$\Rightarrow T'=2\pi\sqrt{\frac{I}{M'B_H}}\Rightarrow T'=\frac{T}{3}=\frac{150}{3}=50\text{ms}$$

30. (5)

$$T=2\pi\sqrt{\frac{r^3}{GM}}$$

$$r=R+h=R+\frac{R}{4}$$

$$GM=gR^2$$

$$T=\frac{2\pi\left(R+h\right)}{R}\sqrt{\frac{R+h}{g}}; \quad T=\frac{2\pi\left(R+\frac{R}{4}\right)}{R}\sqrt{\frac{R+\frac{R}{4}}{g}}$$

$$=2\pi\left(\frac{5}{4}\right)^{\frac{3}{2}}\sqrt{\frac{R}{g}}$$

SECTION-II (CHEMISTRY)

31. (2)

$$\frac{\text{P}^0-\text{P}_\text{S}}{\text{P}_\text{S}}=\frac{\text{n}}{\text{N}}$$

$$\frac{10-5}{5}=\frac{9\times200}{\text{M}\times20}$$

M = 90 amu

32. (3)

For adiabatic free-expansion of an ideal gas:

q = 0, W = 0, $\therefore$ ΔU = 0, $\therefore$ ΔT = 0

But free-expansion is an irreversible process

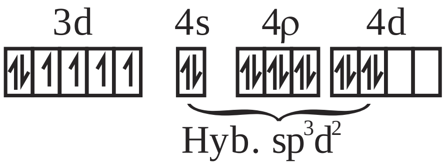
$$\therefore \ \Delta S_{sys}\neq\frac{q_{sys}}{T}\neq\frac{0}{T}\neq0$$

In fact ΔS_{sys} = +ve

33. (3)

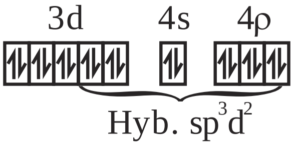
$[Fe(H_2O)_6]^{2+}$ and $[Fe(CN)_6]^{4-}$ differ in magnetic moment and colour.

Fe^{2+} in $\left[Fe(H_2O)_6\right]^{2+}$



colour: Pale green μ = 4.9 B.M.; octahedral geometry

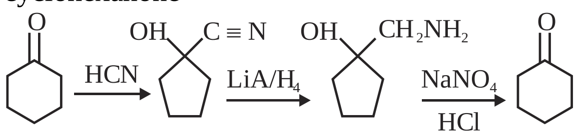
Fe^{2+} in $\left[Fe(CN)_6\right]^{4-}$



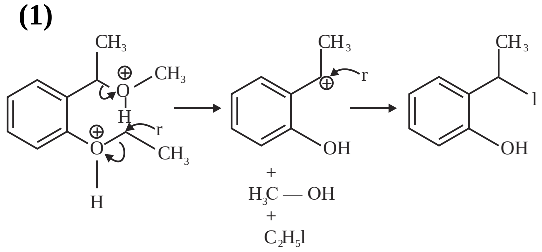
Colour: Yellow; μ = 0; octahedral.

34. (1)

Here the reaction occurs as follows to give cyclohexanone



35. (1)



36. (2)

$\Delta T_b = T_bs - T_bo$

= 373.52 – 373 K = 0.52 K

$\Delta T_b = K_b \times m$

$$0.52 = 0.52 \times \frac{W_{solute} \times 1000}{150 \times W_{solvent}}$$

$$\frac{W_{solute}}{W_{solvent}} = 0.15$$

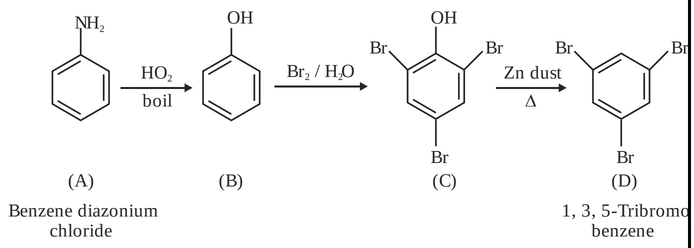
$$\text{Now } \frac{w}{W}\% = \frac{W_{solute}}{W_{solute} + W_{solvent}} \times 100$$

$$= \frac{W_{solute}}{W_{solute} + \frac{W_{solute}}{0.15}} \times 100 \simeq 13\%$$

37. (2)

On going down in group-15, size of central atom increases which leads to increase in the bond length of E-H bond (E = N, P, As, Sb, Bi) and bond strength decreases resulting in decrease in thermal stability of hydrides down the group.

38. (3)



39. (1)

In case of isoelectronic species as $\frac{Z}{e}$ ratio increases, ionic size decreases. Hence size order is

N^{3-} > O^{2-} > F^-

(1.71 Å) (1.40 Å) (1.36 Å)

40. (3)

$H_3BO_3 + H_2O \rightleftharpoons [B(OH)_4]^- + H^2$

$HBF_4 \rightarrow H^+ + BF_4^-$

41. (4)

Option (1) is correct.

Option (2) is correct.

Option (3) is correct.

Option (4) is incorrect.

Rhombic sulphur transforms to monoclinic sulphur when heated above 369 K.

42. (2)

Prussian blue colour is obtained due to the formation of $\left[Fe_4\left(Fe(CN)_6\right)_3\right]$

43. (1)

meq. of $FeSO_4\,(NH_4)_2\,SO_4\,6H_2O$

= meq. of $KMnO_4$(n = 1)

$$\frac{W}{392} \times 1 \times 1000 = 0.1 \times 50;$$

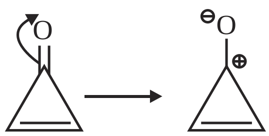
W = 1.96g

Hence, % purity of Mohr's salt

$$= \frac{1.96}{2.5} \times 100 = 78.4\%$$

44. (4)

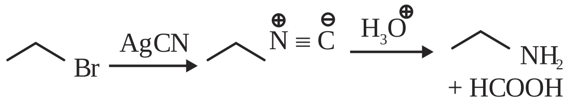
Compound is aromatic in ionic form



4n+ 2 = 2

n = 0, Aromatic

45. **(3)**



46. **(3)**

$$P.E. = \frac{1}{4\pi\varepsilon_0}\frac{(+Ze)(-e)}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(+2e)(-e)}{r} = -\frac{e^2}{2\pi\varepsilon_0 r}$$

47. **(3)**
AB: Isobaric expansion
BC: Isothermal expansion
CD: Isochoric process
DA: Isothermal compression

48. **(2)**
Lattice energy order
$BeF_2 > LiF$

49. **(3)**
$T\ell^{+3} I_3 \rightarrow TI^{+1} + I_2$

50. **(4)**

$$\underset{(S_2)}{2K_2Cr_2O_7} \xrightarrow{\Delta} O_2 + 2K_2Cr_2O_4 + Cr_2O_3$$

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$$

51. **(4)**
Histidine, Valine, Lysine, Methionine

52. **(4)**
$t_{1/2}$ = 3 hours, n = T/$t_{1/2}$ = 18/3 = 6

$$N = N_0\left(\frac{1}{2}\right)^6$$

$$= 256 \times \frac{1}{8} \times \frac{1}{8} = 4g$$

53. **(2)**
DOU = 1
Only aldehydes give 2^0 alocohol
2 aldehydes will give 2^0 alcohols on reaction with CH_3MgBr.

54. **(6)**

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

Mole fraction $\frac{1-\alpha}{1+\alpha}$ $\frac{2\alpha}{1+\alpha}$

$$K_p = \frac{\left[P_{NO_2}\right]^2}{P_{N_2O_4}}$$

$$\Rightarrow K_p = \frac{4\alpha^2}{1-\alpha^2}p$$

For $2NO_2 \rightleftharpoons N_2O_4$;

$$K_p = \frac{0.96}{0.16} = 6$$

55. **(4)**
$\Delta G° = nFE°$

$$-n = \frac{193 \times 10^3}{96500 \times 0.25} = 8$$

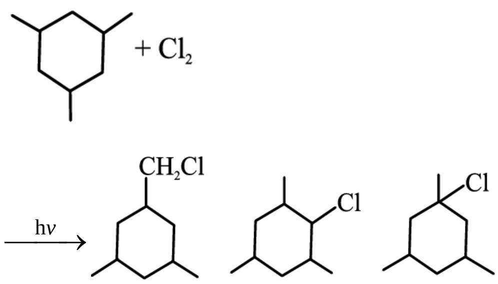
So no of moles of M^+ oxidized by 1 moles of

$$x = \frac{8}{2} = 4$$

56. **(6)**
(i) $H_2C = CH - CH_2 - CH_3$
(ii) $H_3C - \underset{\displaystyle CH_3}{\underset{|}{CH}} - HC = CH_2$
(iii) $H_3C - HC = CH - CH_2 - CH_3$
(iv) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{C}} = CH - CH_3$
(v) $CH_2 = \underset{\displaystyle CH_3}{\underset{|}{C}} - CH_2 - CH_3$

57. **(3)**
PF_3, CF_4 and BeF_2 have all the possible B. A. identical.

58. **(50)**



(A) (B) (C)

Relative reactivity of 1º:2º:3º H towards photochlorination as calculate by above % yield is 1:3.8: 4.5. So, yield of A, B and C will be ratio 9×1: 6×3.8:3×4i.e., 9:22.8:13.5
So, major product is B and its % yield is

$$\frac{22.8}{15.3} \times 100 = 50.33$$

59. **(10)**
$CaOCl_2 \equiv Cl_2 \equiv I \equiv S_2O_3^{2-}$
mEq ≡ mEq ≡ mEq ≡ mEq
Weight of Cl_2 = mEq × 10^{-3} × Ew(Cl_2)

$$= 2 \times 10^{-3} \times \frac{71}{2}$$

= 0.07g

% of available Cl_2 = $\dfrac{\text{Weight of } Cl_2}{\text{Weight of } CaOCl_2} \times 100$

$= \dfrac{0.071 \times 100}{0.71} = 10\%$

60. **(2)**

$(M) = \left(\dfrac{16}{5.6}\right)(22.4)$

M = 64

64 = 32 + 16x

x = 2

SECTION-III (MATHEMATICS)

61. **(4)**

$[f(x) + [f(x)]] = 2\cos x \Rightarrow [f(x)] = \cos x$

$f(x) = \dfrac{1}{4}[\sin x + [\sin x + [\sin x + [\sin x]]]] = [\sin x]$

$\Rightarrow [\sin x] = \cos x$

Number of solution in $[0, 2\pi]$ is 0

Hence total solution is 0.

$\therefore$ both are periodic with period 2π

62. **(2)**

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f''(-1) = 0$

$-6a + 2b = 0$

$\Rightarrow \quad b = 3a$

$f'(1) = 0$

$3a + 2b + c = 0$

$c = -9$

$f(1) = -10$

$-5a + d = -10$ ….(i)

$f(-1) = 6$

$11a + d = 6$ ….(ii)

$a = 1, d = -5, b = 3, c = -9$

$f(x) = x^3 + 3x^2 - 9x - 5$

63. **(1)**

$f'(x) = f(x)$

Integrating $\log f(x) = x + k$

$f(x) = e^{x+k}$

$f(0) = 1$

$1 = e^0 . e^k$

$k = 0$

$\therefore f(x) = e^x$

$g(x) = x - f(x) = x - e^x$

$I = \int_0^1 e^x (x - e^x) dx = \int_0^1 e^x x dx - \int_0^1 e^{2x} dx$

$= e - (e-1) - \dfrac{e^2}{2} + \dfrac{1}{2} = \dfrac{3}{2} - \dfrac{e^2}{2} = \dfrac{3 - e^2}{2}$

64. **(2)**

Taking single digit $\to$ 444444 $\to \dfrac{6!}{6!} = 1$

Taking two digit $\to$

(4, 5) 444555 (4, 9) 444999

$\dfrac{5!}{3!2!} = 10$ $\quad$ $\dfrac{5!}{3!2!} = 10$

Taking three digit

$4, 5, 9, 4, 4, 4 \Rightarrow \dfrac{5!}{3!} = 20$

$4, 5, 9, 5, 5, 5 \Rightarrow \dfrac{5!}{4!} = 5$

$4, 5, 9, 9, 9, 9 \Rightarrow \dfrac{5!}{4!} = 5$

$4, 5, 9, 4, 5, 9 \Rightarrow \dfrac{5!}{2!2!} = 30$

Total = 81

65. **(2)**

$$\int \frac{(x^2+1)}{(x^4 - x^2 + 1)\cot^{-1}\left(x - \frac{1}{x}\right)} dx =$$

$$\int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2} - 2 + 1\right)\cot^{-1}\left(x - \frac{1}{x}\right)} dx$$

$$= \int \frac{dt}{(t^2+1)\cot^{-1} t}$$

Put $x - \dfrac{1}{x} = t \Rightarrow \left(1 + \dfrac{1}{x^2}\right) dx = dt$

$$= \int \frac{-du}{u}$$

Let $\cot^{-1} t = u$

$\Rightarrow \; -\dfrac{1}{(1+t^2)} dt = du = -\ln|u| + c$

$= -\ln\left|\cot^{-1}\left(x - \dfrac{1}{x}\right)\right| + c$

66. **(2)**

$A^2 = 0$, $A^3 = A^4 = ... = A^n = 0$

then $A(I \pm A)^n = A(I + nA)$

$= A \pm nA^2 = A$

67. **(4)**

$$f(x)=\prod_{n=1}^{50}(x-n)^{n(51-n)}$$

$$\ln(f(x)) = \sum_{n=1}^{50} n(51-n)\ln(x-n)$$

Differentiating both side w.r.t. x

$$\frac{f'(x)}{f(x)}=\sum_{n=1}^{50}\frac{n(51-n)}{x-n}$$

$$\Rightarrow \frac{f'(51)}{f(51)}=1275$$

68. **(1)**

$\sin^2\theta = (x-2)^2$

$0 \le (x-2)^2 \le 1$

$-1 \le x-2 \le 1$

$1 \le x \le 3$

69. **(1)**

Vector $\vec{c}$ is perpendicular to

$\vec{a}=3\hat{i}+2\hat{j}+\hat{k}$ & $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

is $\vec{c}=\vec{a}\times\vec{b}=4\hat{i}-8\hat{j}+4\hat{k}$

sum of DC's is 0

70. **(3)**

Let, $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}, \vec{c}=-\lambda\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b}+\vec{c}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Given, projection of $\vec{a}$ on $(\vec{b}+\vec{c})$ is 1

$$\therefore \quad 1=\frac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}$$

$$\Rightarrow \quad \frac{2-\lambda+12-2}{\sqrt{(2-\lambda)^2+36+4}}=1$$

$\Rightarrow \quad (12-\lambda)^2 = (2-\lambda)^2 + 40$

$\Rightarrow \quad 20\lambda = 100$

71. **(3)**

Equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{b^2}=1$ and $ae = 3$

We know that $a^2e^2 = a^2 + b^2$

$9 = 4 + b^2 \Rightarrow b^2 = 5$

Hence, equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$

72. **(1)**

$$A=\int_0^2\left(2^x-\left(2x-x^2\right)\right)dx$$

$$=\left[\frac{2^x}{\ell n2}-x^2+\frac{x^3}{3}\right]_0^2=\left(\frac{4}{\ell n2}-4+\frac{8}{3}\right)-\left(\frac{1}{\ell n2}\right)$$

73. **(1)**

Only symmetric

74. **(3)**

$$x=e^{xy\frac{dy}{dx}}$$

$$\Rightarrow \int y\,dy=\int\frac{\ln x}{x}dx$$

$$\Rightarrow y=\pm\sqrt{(\ln x)^2+c}$$

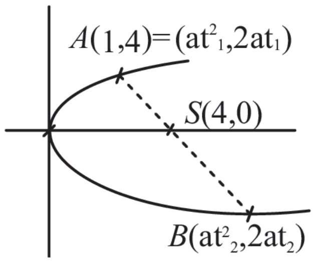
75. **(2)**

$$\tan(\theta_1+\theta_2+\theta_3)=\frac{\sum\tan\theta_1-\prod\tan\theta_1}{1-\sum\tan\theta_1\tan\theta_2}$$

$$=\frac{p+1-q}{1-(q-p)}=1$$

76. **(3)**

(P)



$A(1, 4) \Rightarrow 2.4.t_1 = 4$

$$\Rightarrow \quad t_1=\frac{1}{2}$$

$$\therefore \quad \text{Length of focal chord} = a\left(t+\frac{1}{t}\right)^2$$

$$\lambda=4\left(\frac{1}{2}+2\right)^2=4.\frac{25}{4}=25$$

(Q) $x \in [0, 1) \Rightarrow f(x) = 0$

$x\in\left[1,\sqrt{2}\right)\Rightarrow f(x)=1-1=0$

$x\in\left[\sqrt{2},\sqrt{3}\right)\Rightarrow f(x)=2-1=1$

$x\in\left[\sqrt{3},2\right)\Rightarrow f(x)=3-1=2$

(R) $y=\frac{x+2}{2x^2+3x+6}$

$\Rightarrow y\in\left[-\frac{1}{13},\frac{1}{3}\right]$

(S) $y = e^{-x}\cos x$

$\Rightarrow y_1 = -e^{-x}\sin x - e^{-x}\cos x = -e^{-x}\sin x - y$

$y_2 = -e^{-x}\cos x + e^{-x}\sin x - y_1$

$\Rightarrow y_4 + 4y = 0$

$\Rightarrow k = 2$

77. (4)

Put $x^2 = t,\ \frac{1}{2}\int e^{t^2+t}(1+t+2t^2)dt$

$\frac{1}{2}\int e^{t^2+t}(t(2t+1)+1)\,dt = \frac{1}{2}e^{t^2+t}.t$

78. (3)

$$\lim_{x\to0^+}\left(\ell n\left(\frac{\sin^4 x}{e^2x^4+x^5}\right)\right) = \lim_{x\to0^+}\ell n\left(\frac{1}{e^2+x}\right) = -2$$

79. (4)

Line is $\frac{x}{a}+\frac{y}{b}=1$ passes through (2, 2)

$\frac{2}{a}+\frac{2}{b}=1$ and $-\frac{1}{2}ab = A \Rightarrow \frac{2}{b}=\frac{-a}{A}$

$\frac{2}{a}-\frac{a}{A}=1 \Rightarrow 2A-a^2 = aA \Rightarrow x^2+Ax-2A=0$

80. (3)

$$l = \int_{-2}^{-1}[x[1+(-1)]+1]dx + \int_{-1}^{1}[x[1+0]+1]dx = 2$$

81. (12)

Let $y = \tan^{-1}\left(1+\frac{1-\sqrt{3}}{x^4+\sqrt{3}}\right)$

$\Rightarrow\quad y \in \left[\tan^{-1}\left(1+\frac{1-\sqrt{3}}{\sqrt{3}}\right), \tan^{-1}1\right)$

$\Rightarrow\quad y \in \left[\frac{\pi}{6},\frac{\pi}{4}\right)$

$a+b = \frac{5\pi}{12}$

82. (54)

The given lines will be parallel to lines

$$\lambda x^2 - 12xy + y^2 = 0$$

So, $\left(\frac{y}{x}\right)^2 - 12\left(\frac{y}{x}\right)+\lambda = 0$

$m + m^2 = 12$ …(1)

Which gives $m = 3$ or -4

and $mm^2 = \lambda$ …(2)

From (2) $\lambda = 27$ or -64

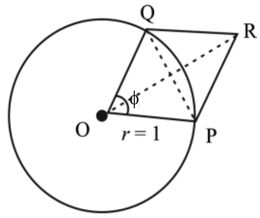
Hence product of all possible value of $\lambda = (-32)\ (54)$

83. (2)

Area of the parallelogram outside the circle

= Area of parallelogram – Area of sector OPQ

$= \left(\frac{1}{2}\sin\phi\right)2 - \frac{1}{2}\phi = \sin\phi - \frac{\phi}{2} = f(\phi)$



84. (13)

In determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, position of a, b, c, d can be filled in $2^4 = 16$ ways. Value of determinant negative when $a = 0$, $d = 1$, $b = 1$, $c = 1$ or $a = 1$, $d = 0$, $b = 1$, $c = 1$ or $a = 0$, $d = 0$, $c = 1$, $b = 1$ total 3 ways

$P = 1 - \frac{3}{16} = \frac{13}{16} \Rightarrow 16P = 13$

85. (64)

$8\,[32 + 15d] = (16 + 15d)^2$

$\Rightarrow\quad 225d^2 + 360d = 0$

$\Rightarrow\quad d = 0,\ \frac{-8}{5}$

$\Rightarrow\quad 25d^2 = 64$

86. (2)

$$\tan\left(\sin^{-1}\left(\frac{2}{\sqrt{4+x^2}}\right)\right) = \frac{2}{|x|} \Rightarrow \lambda = 2$$

87. (3)

$$\int_0^{1/2} e^x\left(\frac{\sqrt{1-x^2}}{1-x}+\frac{1}{\sqrt{1-x^2}\,(1-x)}\right)$$

$$= \left[\frac{e^x\sqrt{1-x^2}}{1-x}\right]_0^{1/2} = \sqrt{3e}-1$$

88. (2)

$PS + PS' = 2a$

$\Rightarrow\quad \sqrt{8}+2 = 2a \Rightarrow a = \sqrt{2}+1$

$SS' = 2ae \Rightarrow\ 2 = 2\left(\sqrt{2}+1\right)e \Rightarrow e+1 = \sqrt{2}$

89. **(32)**

$zw = |z|^2 \Rightarrow |z|\,|w| = |z|^2$

$\Rightarrow \quad |w| = |z|$

Let $z = re^{i\theta} \Rightarrow w = re^{-i\theta}$

$\Rightarrow \quad w = \bar{z} \Rightarrow |z-\bar{z}| + |z+\bar{z}| = 8$

$|x| + |y| = 4 \Rightarrow \text{Area} = 4\left(\frac{1}{2}\cdot 4\cdot 4\right) = 32$

90. **(18)**

$$\sum_{r=1}^{5} {}^{20}C_{2r-1} = {}^{20}C_1 + {}^{20}C_3 + \ldots + {}^{20}C_9$$

$$= \frac{1}{2}\left[\,{}^{20}C_1 + {}^{20}C_3 + \ldots. + {}^{20}C_9 + {}^{20}C_{11} + \ldots.\,{}^{20}C_{19}\right]$$

$$= \frac{1}{2}\left[2^{19}\right] = 2^{18}$$