All India Test Series (JEE–2024)

AVJLM1/04

Test- 04

Lakshya JEE 2024

DURATION : 180 Minutes | DATE : 31/12/2023 | M. MARKS : 300

ANSWER KEY

PHYSICS

1.	(1)
2.	(3)
3.	(2)
4.	(3)
5.	(2)
6.	(2)
7.	(4)
8.	(2)
9.	(4)
10.	(1)
11.	(4)
12.	(2)
13.	(4)
14.	(3)
15.	(3)
16.	(1)
17.	(2)
18.	(1)
19.	(2)
20.	(2)
21.	(1)
22.	(4)
23.	(25)
24.	(1)
25.	(360)
26.	(0)
27.	(1)
28.	(10)
29.	(4)
30.	(100)

CHEMISTRY

31.	(3)
32.	(3)
33.	(3)
34.	(1)
35.	(1)
36.	(2)
37.	(3)
38.	(2)
39.	(2)
40.	(2)
41.	(1)
42.	(2)
43.	(3)
44.	(4)
45.	(2)
46.	(1)
47.	(3)
48.	(2)
49.	(1)
50.	(2)
51.	(7)
52.	(8)
53.	(5)
54.	(306)
55.	(2)
56.	(4)
57.	(3)
58.	(5)
59.	(2)
60.	(1)

MATHEMATICS

61.	(2)
62.	(1)
63.	(1)
64.	(2)
65.	(2)
66.	(3)
67.	(1)
68.	(2)
69.	(1)
70.	(1)
71.	(1)
72.	(4)
73.	(2)
74.	(1)
75.	(4)
76.	(4)
77.	(2)
78.	(4)
79.	(4)
80.	(3)
81.	(0)
82.	(0)
83.	(8)
84.	(25)
85.	(0)
86.	(40)
87.	(35)
88.	(199)
89.	(3)
90.	(30)

SECTION-I (PHYSICS)

1. **(1)**

Here, $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference $= 100\,\text{V}$

$Z = R \quad (\because X_L = X_C)$

$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{100}{50} = 2\,\text{A}$

2. **(3)**

At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ *and* q_2. Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

3. **(2)**

$\text{KE}_{\lambda_1} = \frac{hc}{\lambda_1} - \Psi = e\Delta V$

$\text{KE}_{\lambda_2} = \frac{hc}{\lambda_2} - \Psi = 2e\Delta V$

$\Rightarrow 3\left(\frac{hc}{\lambda_1} - \Psi\right) = \frac{hc}{\lambda_2} - \Psi$

$\Rightarrow \Psi = hc\left(\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right)$

$\Rightarrow \text{KE}_{\lambda_3} = \frac{hc}{\lambda_3} - hc\left[\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right]$

$= hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$

$e\Delta V = hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$

$\Delta V = \frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$

4. **(3)**

Velocity of light waves in material is

$v = n\lambda$ ...(i)

Refractive index of material is

$\mu = \frac{c}{v}$ ...(ii)

where c is speed of light in vacuum or air.

Or $\mu = \frac{c}{n\lambda}$ ...(iii)

Given $n = 2 \times 10^{14}$ Hz

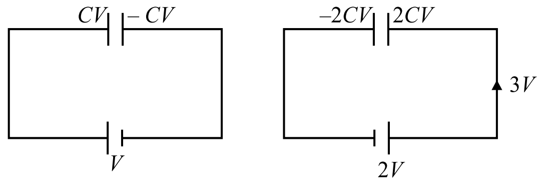
$\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m

$c = 3 \times 10^8$ m/s

Hence, from Eq. (iii), we get

$\mu = \frac{3\times10^8}{2\times10^{14}\times5000\times10^{-10}} = 3.00$

5. **(2)**



Heat = $6CV^2 - \left\{\frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2\right\}$

6. **(2)**

P_{output} = 1500 W

$\varepsilon_0 = \sqrt{\frac{2P}{A\varepsilon_0 C}}$

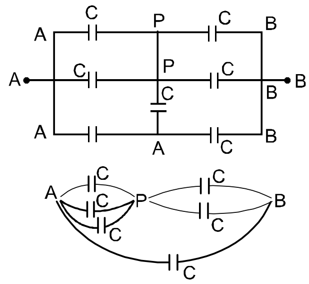
$\Rightarrow \sqrt{\frac{2\times1500}{4(3.14)(9)(8.85\times10^{-12})(3\times10^8)}}$

$\Rightarrow 100\ V/m$

7. **(4)**

Saturation current is inversely proportional to the square of distance of cathode from point source

8. **(2)**



Get $C_{eq} = \frac{11c}{5}$

Charge flow = $C_{eq}\,\varepsilon = \frac{11\,c\varepsilon}{5}$.

9. **(4)**

E.m.f =6 V; total resistance = 6 Ω, I =6/6=1 A

For the direction of current, look at the direction of e.m.f of the cell of 10 V

10. **(1)**

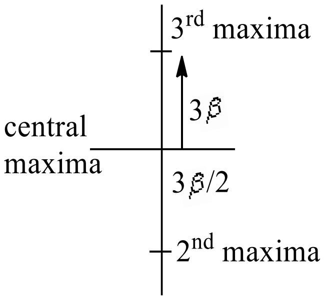
Frequency remains unchanged as electromagnetic waves pass from one medium to another.

11. **(4)**

$d = 0.5$ mm and $D = 0.5$ m

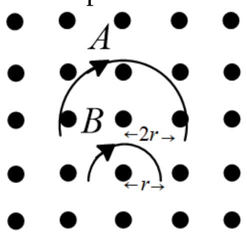
Separation $= 3\beta + 1.5\beta = 4.5\beta$

$= 4.5 \times \frac{\lambda D}{d} = 2.22\,\text{mm}$



12. **(2)**

When a charged particle is moving at right angle to the magnetic field, then a force acts on it which behaves as a centripetal force and moves the particle in circular path



$\therefore \frac{m_A v_A^2}{2r} = q v_A B \; \therefore \frac{m_A v_A}{2r} = qB$

Similarly, for second particle moving with half radius as compared to the first one, we have

$\frac{m_B v_B}{r} = qB \Rightarrow \frac{m_A v_A}{2r} = \frac{m_B v_B}{r} \;\Rightarrow m_A v_A = 2 m_B v_B$

$\Rightarrow m_A v_A = m_B v_B$

$\therefore$ Correct option is (2)

13. **(4)**

Focal length of plano convex lens is

$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$ or $f = 20\,\text{cm}$

If point object O is placed at a distance of 20 cm from the plano-convex lens, rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of b

14. **(3)**

Path difference $= (\mu_2 - \mu_1)t = 12480\,\mathring{A}$

For maxima,

$n\lambda = 12480\,\mathring{A}$

$\lambda_1 = 12480\,\mathring{A}$

$\lambda_2 = 6240\,\mathring{A}$

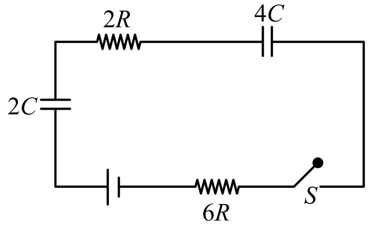
$\lambda_3 = 4160\,\mathring{A}$

$\lambda_4 = 3120\,\mathring{A}$

Therefore, only $6240\,\mathring{A}$ and $4160\,\mathring{A}$ exist in the spectrum

15. **(3)**

$R_{eq} = 8R$



$C_{eq} = \frac{4C}{3}$

$\tau = R_{eq} \times C_{eq} = \frac{32CR}{3}$

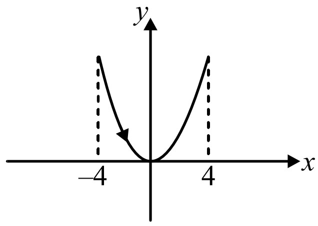
16. **(1)**

$F = iB.\ell = 4 \times 0.04 \times 8$

$F = 1.28$ newton

$ma = 1.28$

$a = \frac{1.28}{0.2} = \frac{12.8}{2} = 6.4\,\text{m/s}^2$



17. **(2)**

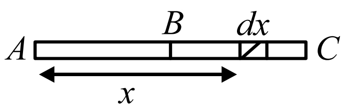
Initial flux: $\phi_i = -\int_a^{2a} \frac{\mu_0 Ia}{2\pi}\frac{dx}{x} = -\frac{\mu_0 Ia}{2\pi}\text{In}2$

Final flux : $\phi_f = \frac{\mu_0 Ia}{2\pi}\text{In}2$

Change flown : $q = \frac{\Delta\phi}{r} = \frac{\phi_f - \phi_i}{r} = \frac{\mu_0 Ia}{\pi r}\text{In}2$

18. **(1)**

$di = \frac{\lambda dx}{\frac{2\pi}{\omega}} = \frac{\omega\lambda dx}{2\pi}$



$dB = \frac{\mu_0 di}{2x}$

$dB = \frac{\mu_0}{2x}.\frac{\omega\lambda}{2\pi}dx$

$\int dB = \frac{\mu_0\omega\lambda}{4\pi}\int_{\ell/2}^{\ell}\frac{dx}{x} = \frac{\mu_0\omega\lambda}{4\pi}\ln 2$

19. (2)

Even if ammeter is non-ideal, its resistance should be small and net parallel resistance is less than the smallest individual resistance.

$\therefore$ R_{net} < resistance of ammeter in the changed situation. Hence, net resistance of the circuit will decrease. So, main current will increase. But maximum percentage of main current will pass through ammeter (in parallel combination) as its resistance is less. Hence, reading of ammeter will increase.

Initial voltmeter reading = emf of battery

Final voltmeter reading = emf of battery

− potential drop across shown resistance.

Hence, voltmeter reading will decrease.

20. (2)

$i_1 = i_2$ or i is same at both sections.

$$A_1 < A_2$$

(A) Current density $= \frac{i}{A} \propto \frac{1}{A}$

(C) Conductance = $\frac{1}{R} = k = \frac{\sigma A}{l}$

For unit length $l = 1$, $k \propto A$

(D) And (*b*) *E* or potential difference per unit length = (*i*) (Resistance per unit length)

$$= (i)\left(\frac{\rho}{A}\right) \propto \frac{1}{A}$$

21. (1)

$KE = 100 + 50 = 150\text{eV}$

$v = 150$ volt

$$\lambda = \sqrt{\frac{150}{V}}$$

$\lambda = 1\text{Å}$

22. (4)

For shortest wavelength in Balmer series,

$n_1 = 2; n_2 = \infty$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{\infty}\right] \text{or } \lambda = \frac{4}{R}$$

For shortest wavelength in Brackett series,

$n_1 = 4; n_2 = \infty$

$$\therefore \frac{1}{\lambda'} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$$

or $\lambda' = \frac{16}{R} = 4 \times \frac{4}{R} = 4\lambda$

23. (25)

$$I = \frac{I_0}{2}\left(\frac{1}{4}\right) \Rightarrow \frac{I}{I_0} = \frac{1}{8}$$

Or $\frac{100}{8}\% = 12.5\%$

24. (1)

Electric field will remain the same, because electric field due to surface charge distributed uniformly will be zero at any point inside the sphere

25. (360)

$$I = \frac{V}{Z}$$

$V_C = IX_C = 360$

26. (0)

By symmetry, we see that the current in the left and right arms should be the same. It means no current should flow from A to B

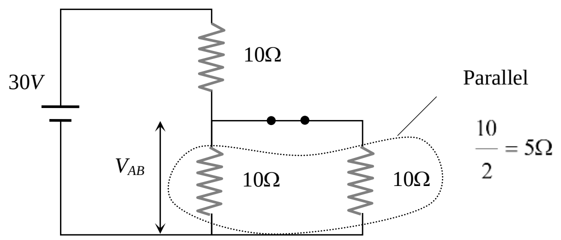
27. (1)

From conservation of linear momentum, both the particles will have equal and opposite momentum. The de Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow \lambda_1 / \lambda_2 = 1$$

28. (10)

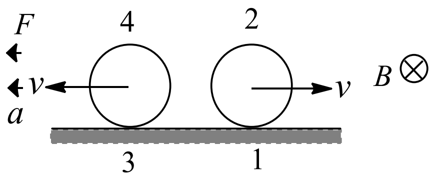
Diode is in forwards biasing hence the circuit can be redrawn as follows



$$V_{AB} = \frac{30}{(10+5)} \times 5 = 10V$$

29. (4)

$$V_2 - V_1 = \frac{1}{2}B\omega(2R)^2 = 2BR(\omega R) = 2BRv$$



$V_3 - V_4 = 2BRv$ But $V_1 = V_3$

$\Rightarrow V_2 - V_4 = 4BRv$

30. (100)

$$E_v = \sqrt{120^2 + 160^2} = 200\,\text{V}$$

31. **(3)**

Reactions that occurs at anode

$Zn \rightarrow Zn^{2+} + 2e^-$

At cathode

$Br^2 \rightarrow 2Br^- + 2e^-$

$\therefore$ $[Zn^{2+}]$ and $[Br^-]$ increases and $[Br_2]$ and mass of Zn rod decreases with time.

32. **(3)**

(1) $NH_4^+ + OH^- \longrightarrow NH_3\uparrow + H_2O$.

(2) The statement is correct ; non-chiral because there is mirror plane through the metal bisecting the dien ligand.

(3) $4Cl^-$ (s) + $Cr_2O_7^{2-}$ (s) + $6H^+$ (conc.) $\longrightarrow$ $2CrO_2Cl_2\uparrow$ (deep red) + $3H_2O$ (l)

(4) VCl_3; ${}_{23}V^{3+}$;$[Ar]^{18}3d^2$ = 2 unpaired electrons.

VO^{2+} ; V^{4+} ; $[Ar]^{18}$ $3d^1$ = 1 unpaired electron.

$[VO_4]^{3-}$; V^{5+} ; $[Ar]^{18}$ $3d^0$

= no unpaired electron.

$[V(H_2O)_6]^{2+}$; V^{2+} : $[Ar]^{18}$ $3d^3$ = 3 unpaired electrons.

33. **(3)**

$2H^+ + 2e^- \rightarrow H_2$

$\therefore$ RP $= 0.0 - \dfrac{0.059}{2}\log\dfrac{P_{H_2}}{[H^+]^2}$

34. **(1)**

Rate is governed by slowest step

$A + B_2 \xrightarrow{k_1} AB + B$

$r = k_1[A][B_2]$ .... (i)

From $A_2 \overset{k_c}{\rightleftharpoons} A + A$

$k_C = \dfrac{[A]^2}{[A_2]}$ . ... (ii)

$[A] = \sqrt{k_c}\,[A_2]^{1/2}$

$r = k_1\sqrt{k_c}\,[A_2]^{1/2}[B_2]$

order is $= \dfrac{1}{2} + 1 = \dfrac{3}{2}$

35. **(1)**

(a) (F); As the size of halogen atom increases crowding on Si atom will increase, hence, tendency of attack of Lewis base decreases.

(b) (T); M.P. of NH_3 is highest due to intermolecular H-bonding in it.

Next lower M.P. will be of SbH_3 followed by AsH_3 due to high mol. wt. of SbH_3.

(c) (F); M.P. and B.P. of increase from PH_3 to SbH_3 via AsH_3 due to increase in mol. wt. NH_3 does not follow this trend due to inter molecular H-bonding.

(d) (T); Value of bond moment decreases.

36. **(2)**

Step-III is the RDS

Overall ΔH would be negative as aromatisation takes place.

37. **(3)**

(i) It shows ionisation, linkage as well as geometrical isomerism.

(ii) $[Co(NH_3)_4(NO_2)_2]^+$ is diamagnetic inner orbital complex with t_{2g}^6 electron configuration.

(iv) $[V(CO)_6]^-$ is diamagnetic with t_{2g}^6 electron configuration.

38. **(2)**

The integrated rate law for the reaction is

$$\log\frac{[A]_0}{[A]} = \frac{k}{2.303}t$$

$$\log[A] = \log[A]_0 - \frac{k}{2.303}t$$

$$\Rightarrow \frac{k}{2.303} = 3.01\times10^3$$

$\Rightarrow k = 6.93 \times 10^3\ s^{-1}$

39. **(2)**

$NaNO_3 + 8[H] \xrightarrow{Zn/NaOH} NaOH + 2H_2O + NH_3$

(1) $NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

(2) $NH_4NO_2 + NaOH \xrightarrow{\Delta} NaNO_2 + NH_3 + H_2O$

(3) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

(4) $2Ca(NO_3)_2 \xrightarrow{\Delta} 2CaO + 4NO_2 + O_2$

40. **(2)**

(2) $Zn^{2+} + 2OH^- \longrightarrow Zn(OH)_2$ (white)

$Zn(OH)_2\downarrow + 2OH^- \longrightarrow [Zn(OH)_2]^{2-}$ (colourless soluble complex)

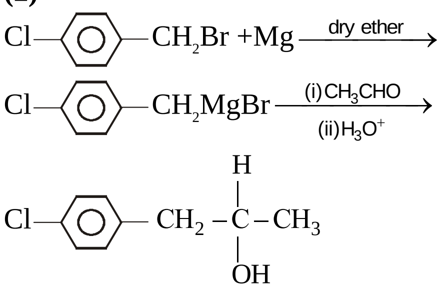
$Zn^{2+} + 2NH_3 + 2H_2O \longrightarrow Zn(OH)_2\downarrow$ (white) $+ 2NH_4^+$

$Zn(OH)_2\downarrow + 4NH_3 \longrightarrow [Zn(NH_3)_4]^{2+} + 2OH^-$

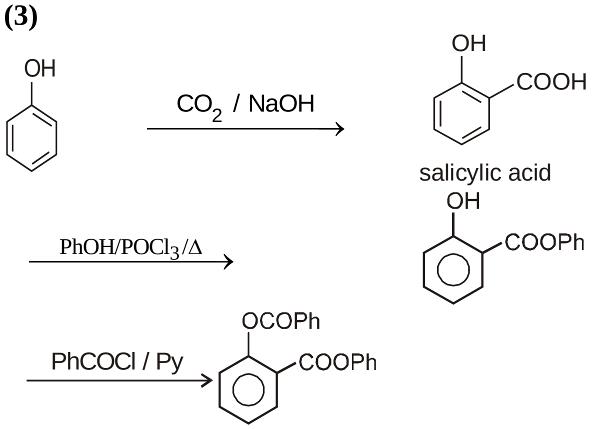
41. **(1)**

Inversion + Inversion = Retention

42. **(2)**



43. **(3)**



44. **(4)**

Cellulose is the polymer of glucose

45. **(2)**

$CH_3-CH_2-C\equiv CH \xrightarrow{Na} CH_3-CH_2-C\equiv CNa$

$\xrightarrow[S_N2]{CH_3-I} CH_3-CH_2-C\equiv C-CH_3$

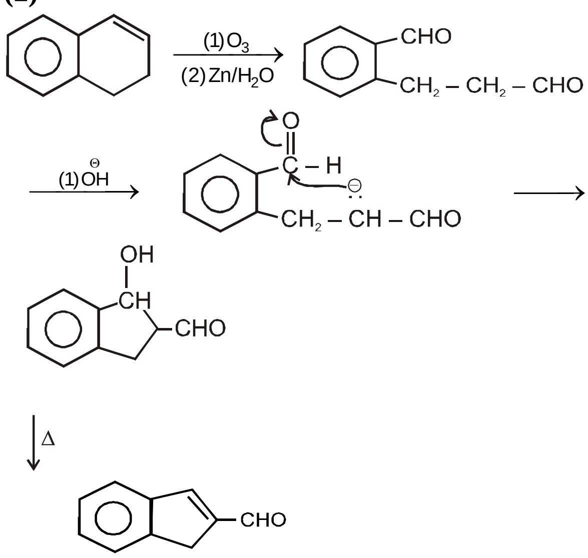
46. **(1)**

In the above substitution reaction if the leaving group is better then rate of reaction is faster.

47. **(3)**

–I group increases acidity.

48. **(2)**

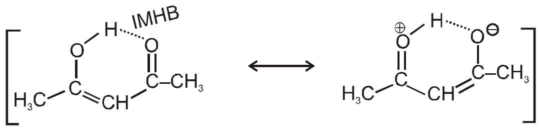


49. **(1)**

Electrophile attack on that ring which have more +M effect.

50. **(2)**

Enol (Q) is stabilised by resonance and hydrogen bond.



51. **(7)**

30% decomposition means that x = 30% of a = 0.30 a

As reaction is of 1st order,

$k = \frac{2.303}{t}\log\frac{a}{a-x} = \frac{2.303}{40\,\text{min}}\log\frac{a}{a-0.30a}$

$= \frac{2.303}{40}\times\log\frac{10}{7}\ \text{min}^{-1}$

$= \frac{2.303}{40}\times 0.1549\ \text{min}^{-1} = 8.918\times 10^{-3}\ \text{min}^{-1}$

For a 1st order reaction,

$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{8.918\times 10^{-3}\ \text{min}^{-1}} = 77.7$ min.

52. **(8)**

$2Al_2O_3\ (\text{melt}) \longrightarrow 2Al_2O_3(s)$, $\Delta G_1^o = -2\times 16$

$2Al_2O_3(s) \longrightarrow 4Al(l) + 3O_2(g)$, $\Delta G_2^o = 2\times 269$

$3C + 3O_2 \longrightarrow 3CO$, $\Delta G_3^o = -3\times 395$

$2Al_2O_3\ (\text{melt}) + 3C \longrightarrow 4Al\ (l) + 3CO_2(g)$,

$\Delta G^o = \Delta G_1^o + \Delta G_2^o + \Delta G_3^o$

$= -32 + 2\times 1269 - 3\times 395 = 1321$ kJ

$\Delta G^o = -nFE^o$

$\Rightarrow 1321\times 10^3 = -12\times E^o\times 96500$

$E^o = -1.14$ volt

53. **(5)**

$K_f = 186°\ \text{cm}^{-1}$

$\Delta T_f = i\times K_f.m$

$3.82 = i\times 1.86\times\frac{5\times 1000}{142\times 45}$

$i = 2.63$

54. **(306)**

$P_T = X_{Heptane}P^o_{Heptane} + X_{Octane}P^o_{Octane}$

$= \frac{0.25}{0.557}\times 75 + \frac{0.307}{0.557}\times 50$

$33.66 + 27.558 = 61.2$ kPa

55. **(2)**

The upper curve represents the vapour phase composition and the lower curve represents liquid phase composition.

Composition in vapour phase corresponds to point Q when liquid is boiling with composition corresponds to point P.

56. **(4)**

Cell reaction:

$2Br^- + 2H^+ \longrightarrow Br_2 + H_2$

Nernst eqn.: $E_{cell} = E^o_{cell} - \frac{0.0591}{2} \log \frac{1}{[Br^{-2}][H^+]^2}$

$\therefore E_{cell} = (0 - 1.08) - \frac{0.0591}{2} \log \frac{1}{(0.01)^2 (0.03)^2}$

$= -1.08 - \frac{0.0591}{2} \log (1.111 \times 10^7)$

$= -1.08 - \frac{0.0591}{2} (7.0457) = -1.08 - 0.208$

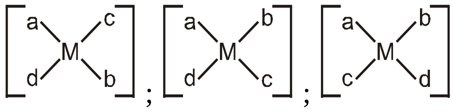
$= -1.288$ V

57. **(3)**

M(abcd)] complex is square planar, so will have 3 geometrical isomers.

(i) (a T b) (c T d); (ii) (a T c) (b T d) ;

(iii) (a T d) (b T c)



58. **(5)**

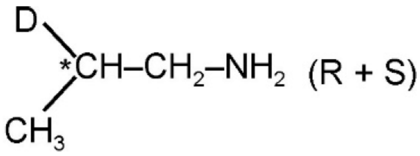
Only gly is optically inactive

x = 5 ; y = 1

59. **(2)**

There will be no intermolecular reaction between amides

Product obtained are



60. **(1)**

$3MnO_4^{2-} + 2H_2O \longrightarrow 2MnO_4^- + MnO_2\downarrow + 4OH^-$

SECTION-III (MATHEMATICS)

61. **(2)**

$\left|\bar{c} - \bar{b} + 2\bar{a}\right|^2 = |\bar{c}|^2 + |\bar{b}|^2 + 4|\bar{a}|^2 - 2\bar{b}\cdot\bar{c} - 4\bar{a}\cdot\bar{b} + 4\bar{c}\cdot\bar{a}$

$\because\ \bar{b}\cdot\bar{c} = 0,\ \frac{\bar{b}\cdot\bar{a}}{|\bar{a}|} = \frac{\bar{c}\cdot\bar{a}}{|\bar{a}|}$

$\left|\bar{c} - \bar{b} + 2\bar{a}\right|^2 = 1 + 1 + 4 = 6$

62. **(1)**

Let $h(x) = f_1(x) - f_2(x) = 2x - 3\sin x + x\cos x$

$h(0) = 0$

$h'(x) = 2 - 2\cos x - x\sin x$

$h''(x) = \sin x - x\cos x$

$h'''(x) = x\sin x$

$h'''(x) > 0 \Rightarrow h''(x) > 0 \Rightarrow h'(x) > 0 \Rightarrow h(x) > 0$

63. **(1)**

Total formed numbers that begin with a odd digit

$= {}^5C_1 \cdot {}^8P_4 = 5(8)(7)(6)(5)$

Total formed numbers that end with a odd digit

$= {}^5C_1 \cdot {}^8P_4 = 5(8)(7)(6)(5)$

Total formed number that begin with an odd digit and also end with an odd digit

$= {}^5C_2 \cdot 2! \cdot {}^7P_3 = 5 \cdot (4)(7)(6)(5)$

Thus total formed numbers that begin with an odd digit or end with an odd digits is equal to $5 \cdot 7 \cdot 6 \cdot 60$

Total formed numbers $= {}^9P_5 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

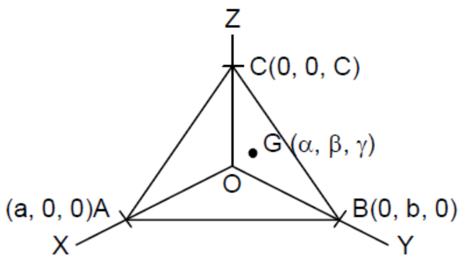
Thus, required probability $= \frac{5}{6}$

64. **(2)**

$\alpha = \frac{a}{4}, \beta = \frac{b}{4}, \gamma = \frac{c}{4}$

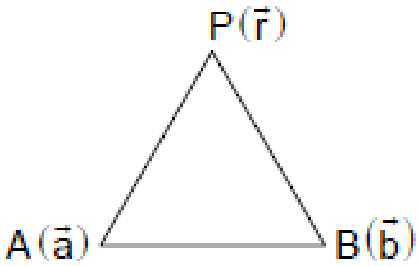
$a = 4\alpha$, $b = 4\beta$, $c = 4\gamma$

Volume of tetrahedron $OABC = \frac{1}{3}\left(\frac{1}{2} \times ab\right) \times c = \frac{abc}{6}$



$\frac{64\alpha\beta\gamma}{6} = 64, \alpha\beta\gamma = 6,$ locus $xyz = 6$

65. **(2)**



$\left|\overrightarrow{PA}\right| = \left|\overrightarrow{PB}\right|$

$|\vec{r} - \vec{a}|^2 = |\vec{r} - \vec{b}|^2$

$|\vec{r}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{r} = |\vec{r}|^2 + |\vec{b}|^2 - 2\vec{b}.\vec{r}$

$|\vec{a}|^2 - |\vec{b}|^2 = 2\vec{r}.\left(\vec{a} - \vec{b}\right)$

$$2\vec{r}.\left(\vec{a}-\vec{b}\right)=|\vec{a}|^2-|\vec{b}|^2$$
$$2\vec{r}.\left(\vec{a}-\vec{b}\right)=\left(\vec{a}+\vec{b}\right)\left(\vec{a}-\vec{b}\right)$$
$$\left(2\vec{r}-\left(\vec{a}+\vec{b}\right)\right).\left(\vec{a}-\vec{b}\right)=0$$

66. **(3)**

Probability that A and B can solve but 'C' is unable to solve

$= \frac{1}{2}.\frac{1}{3}.\frac{3}{4}=\frac{1}{8}$

Probability that A and C can solve but 'B' is unable to solve

$= \frac{1}{2}.\frac{2}{3}.\frac{1}{4}=\frac{1}{12}$

Probability that B and C can solve but 'A' is unable to solve

$= \frac{1}{2}.\frac{1}{3}.\frac{1}{4}=\frac{1}{24}$

Probability that all of them can solve the problem

$= \frac{1}{2}.\frac{1}{3}.\frac{1}{4}=\frac{1}{24}$

Thus required probability that atleast two of them can solve the problem = $\frac{7}{24}$

67. **(1)**

L.H.S.

$$=\begin{vmatrix} 1 & \cos A\cos B+\sin A\sin B & \cos A\cos C+\sin A\sin C \\ \cos B\cos A+\sin A\sin B & 1 & \cos B\cos C+\sin B\sin C \\ \cos C\cos A+\sin A\sin C & \cos C\cos B+\sin C\sin B & 1 \end{vmatrix}$$

$$=\begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix}\times\begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix}=0$$

Hence (1) is the correct answer.

68. **(2)**

$$\lim_{x\to0^+}\frac{\cos^{-1}(1-x)}{\sqrt{x}}=\lim_{x\to0^+}\frac{\sin^{-1}\sqrt{1-(1-x)^2}}{\sqrt{x}}$$
$$=\lim_{x\to0^+}\left(\frac{\sin^{-1}\sqrt{2x-x^2}}{\sqrt{x}}\right)\frac{\sqrt{2x-x^2}}{\sqrt{2x-x^2}}$$
$$=\lim_{x\to0^+}\frac{\sqrt{2x-x^2}}{\sqrt{x}}=\sqrt{2}$$

69. **(1)**

Probability $=\frac{{}^6C_5\,5!\,5!}{10!}=\frac{6!5!}{10!}=\frac{1}{42}$

70. **(1)**

Let I = $\int\frac{3+2\cos x}{(2+3\cos x)^2}dx$

Multiplying Nr. & Dr. by $\text{cosec}^2 x$

$$\Rightarrow I = \int\frac{(3\text{cosec}^2 x+2\cot x\,\text{cosec}\,x)}{(2\,\text{cosec}\,x+3\cot x)^2}dx$$
$$=-\int\frac{-3\text{cosec}^2 x-2\cot x\,\text{cosec}\,x}{(2\,\text{cosec}\,x+3\cot x)^2}dx$$
$$=\frac{1}{2\,\text{cosec}\,x+3\cot x}=\left(\frac{\sin x}{2+3\cos x}\right)+c.$$

Hence (1) is the correct answer.

71. **(1)**

Only reflexive $a = b \Rightarrow \sqrt{3}$ is an irrational number $\left(\sqrt{3},1\right)\in R$ but $\left(1,\sqrt{3}\right)\notin R$ not symmetric $\left(\sqrt{3},1\right),\left(1,2\sqrt{3}\right)\in R$ but $\left(\sqrt{3},2\sqrt{3}\right)\notin R$ not transitive.

72. **(4)**

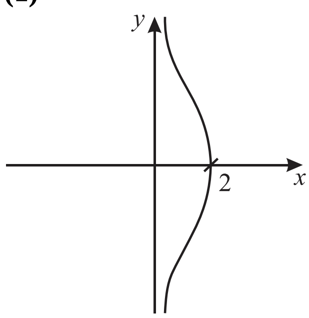
–1 does not belongs to natural number.

73. **(2)**

B = cofactor matrix of A

$|B| = |A|^2$

74. **(1)**



Area $=\int_{-\infty}^{\infty}x\,dy$

$=\int_{-\infty}^{\infty}\frac{2}{1+y^2}dy$

$= 2\pi$

75. **(4)**

Since cos $(-x)$ = cos x

$\therefore$ cos $|x|$ is differentiable for each $x\in R$.

Also, $x^2-3x+2>0 \Rightarrow (x-2)(x-1)>0$

$\Rightarrow x\in(-\infty,1)\cup(2,\infty)$

Similarly, $x^2-3x+2<0 \Rightarrow x\in(1,2)$

If $g(x)=\left(x^2-1\right)|x^2-3x+2|$, then $f(x)$ is not differentiable at points where $g(x)$ is so.

Now,

$g(x)=(x-1)^2(x+1)(x-2), \forall\, x\in(-\infty,1)\cup(2,\infty)$

and $=-(x-1)^2(x+1)(x-2)\ \forall\, x\in(1,\ 2)$

$\therefore$ $g(x)$ is not differentiable at $x = 2$

$\Rightarrow$ $f(x)$ is not differentiable at $x = 2$.

Hence (4) is correct answer.

76. **(4)**

Let $\bar{r} = x\hat{i} + y\hat{i} + z\hat{k}$

$\bar{r}\times(\hat{i}+2\hat{j}-\hat{k})=\hat{i}+\hat{k}$

$-(y+2z)\hat{i}+(x+z)\hat{j}+(2x-y)\hat{k}=\hat{i}+\hat{k}$

Compare

$\bar{r}=x\hat{i}+(2x-1)\hat{j}-x\hat{k}$ for $x \in R$

77. **(2)**

$$\int e^x\left(\frac{x+1}{x+2}+\frac{1}{(x+2)^2}\right)dx=e^x\left(\frac{x+1}{x+2}\right)+c$$

$f(x)=\dfrac{x+1}{x+2}=y$

$yx + 2y = x + 1$

$x=\dfrac{1-2y}{y-1}\Rightarrow y\neq 1$

78. **(4)**

$f'(x) = 3x^2 - \dfrac{3}{x^4} = 0 \Rightarrow x = \pm 1$

$f(x) \in (-\infty, -2] \cup [2, \infty)$

79. **(4)**

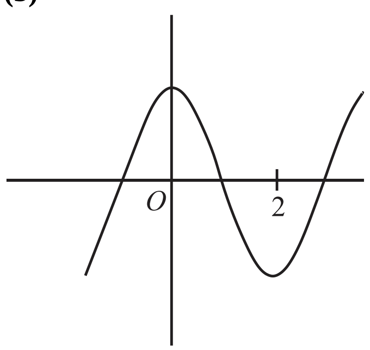
$\dfrac{d}{dx}\left(x\dfrac{dy}{dx}\right)=\ln x\Rightarrow x\dfrac{dy}{dx}=x\ln x-x+c\Rightarrow c=0$

(A (1, 1) and $\dfrac{dy}{dx}=-1$)

$x\dfrac{dy}{dx}=x\ln x-x\Rightarrow dy=(\ln x-1)dx$

$\Rightarrow y=x\ln x-x-x+c_1\Rightarrow c_1=3$

80. **(3)**



$x+\dfrac{a}{x^2}>3$

$x^3-3x^2+a>0$

$f(x)=x^3-3x^2+a$

$f'(x)=3x^2-6x=3x(x-2)$

$f(2) > 0$

$-4 + a > 0$

$a > 4$

81. **(0)**

$f(x) = \sin(\tan^{-1}(\cos(\cot^{-1}x)))$

$x \in R \Rightarrow f(x) \in \left(-\dfrac{1}{\sqrt{2}},\dfrac{1}{\sqrt{2}}\right)$

82. **(0)**

Let $\dfrac{2x-1}{x^2+1}=z \Rightarrow y=f(z)$

$\therefore \quad \dfrac{dy}{dx}=f'(z)\cdot\dfrac{dz}{dx}$

$=\sin z^2\cdot\dfrac{dz}{dx} \qquad (\because f'(z)=\sin z^2)$

$=\sin\left(\dfrac{2x-1}{x^2+1}\right)^2\dfrac{d}{dx}\left(\dfrac{2x-1}{x^2+1}\right)$

$=\sin\left(\dfrac{2x-1}{x^2+1}\right)^2\dfrac{2(1+x-x^2)}{(x^2+1)^2}.$

At $x=\dfrac{1}{2},\dfrac{dy}{dx}=0$

83. **(8)**

Since, nine digit numbers consists of 1, 2, 3, …..9 where sum 1 + 2 + 3 + ….+ 9 = 45 So, it is divisible by 9, to be divisible by 36. This number should also be divisible by 4 So, last two digits must be 12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96 So there are 16 options. Number of favourable event = 16 × 7!

Total number of events = 9!, so probability $p=\dfrac{16\times 7!}{9\times 8\times 7!}=\dfrac{2}{9}$

$9p = 2 \Rightarrow 36p = 8$

84. **(25)**

$y\left(\dfrac{dy}{dx}\right)^2+x\dfrac{dy}{dx}-y\dfrac{dy}{dx}-x=0$

$y\dfrac{dy}{dx}\left(\dfrac{dy}{dx}-1\right)+x\left(\dfrac{dy}{dx}-1\right)=0$

$\left(y\dfrac{dy}{dx}+x\right)\left(\dfrac{dy}{dx}-1\right)=0$

$\therefore$ either $ydy + xdx = 0$ or $dy - dx = 0$

Since the curves pass through the point (3, 4)

$\therefore$ $x^2 + y^2 = 25$ or $x - y + 1 = 0$

85. (0)

$$I_1 = \int_0^{-1} e^{(x+1)^2} dx$$

Let $x + 1 = t$

$dx = dt$

$$I_1 = \int_1^0 e^{t^2} dt$$

$$I_2 = 3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx = 3\int_{1/3}^{2/3} e^{(3x-2)^2} dx$$

Let $2 - 3x = t$

$-3dx = dt$

$$I_2 = -\int_1^0 e^{t^2} dt$$

$I_1 + I_2 = 0$

86. (40)

$\because$ $f(x)$ is surjective

$\therefore$ minimum value of $ax^2 - 2px + 4$ is 1

$\Rightarrow \frac{-D}{4a} = 1 \Rightarrow 4p^2 = 12a$

$\Rightarrow p^2 = 3a$ where $a, p \in N$

For smallest $\frac{20}{3}$ $(a + p)$, $a = 3$, $p = 3$

$\therefore \frac{20}{3}(a + p) = 40$

87. (35)

$-x^2 - 2x + k \geq 0$

$x^2 + 2x - k \leq 0$

$x \in \left[-1-\sqrt{1+k}, -1+\sqrt{1+k}\right]$

$-1+\sqrt{1+k} \geq 5$

$k \geq 35$

88. (199)

$$A^2 = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

So, $(I + A)^{80} = {}^{80}C_0 I + {}^{80}C_1 IA + \ldots.. + {}^{80}C_{80} A^{80}$

$(I + A)^{80} - 80A = I$

$$\begin{bmatrix} a+1 & b+1 \\ c+1 & d-200 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$a = 0$, $b = -1$, $c = -1$, $d = 201$

$a + b + c + d = 199$

89. (3)

Any point on the line is

$P = (6r_1 + 2, 3r_1 + 3, -4r_1 - 4)$.

Direction ratios of the line segment PQ are

$\langle\langle 6r_1 + 3, 3r_1 + 1, -4r - 10 \rangle\rangle$ (where $Q(-1, 2, 6)$),

If 'P' be the foot of altitude drawn from Q to the given line, then

$6(6r_1 + 3) + 3(3r_1 + 1) + 4(4r_1 + 10) = 0$.

$\Rightarrow$ $r_1 = -1$.

Thus, $P = (-4, 0, 0)$

$\therefore$ Required distance $\left|\overline{PQ}\right| = \sqrt{9+4+36} = b$

$b = 7$ units., $\alpha = -4$

$\alpha + b = 3$

90. (30)

$$I_1 + I_2 = \int_0^{\pi} f(x).\sin x\, dx + \int_0^{\pi} f''(x)\sin x\, dx$$

$$= f(x).-\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x. f'(x)dx$$

$$+ \sin x\, f'(x)\Big|_0^{\pi} - \int_0^{\pi} \cos x. f'(x)dx$$

$\Rightarrow f(\pi) + f(0) = 5(given)$

$\Rightarrow f(0) = 5 - f(\pi) = 5 - 2 = 3$