AITS
ALL INDIA TEST SERIES
SELECTION HOGA YAHIN SE

12th JEE Main

AJLM3/02

Test- 02

DURATION : 180 Minutes

DATE : 17/11/2024

M.MARKS : 300

ANSWER KEY

PHYSICS

1.	(1)
2.	(1)
3.	(3)
4.	(2)
5.	(2)
6.	(2)
7.	(1)
8.	(4)
9.	(4)
10.	(1)
11.	(2)
12.	(2)
13.	(1)
14.	(2)
15.	(4)
16.	(3)
17.	(2)
18.	(1)
19.	(1)
20.	(3)
21.	(50)
22.	(10)
23.	(50)
24.	(9)
25.	(11)

CHEMISTRY

26.	(3)
27.	(4)
28.	(3)
29.	(3)
30.	(4)
31.	(2)
32.	(3)
33.	(3)
34.	(1)
35.	(4)
36.	(2)
37.	(1)
38.	(3)
39.	(1)
40.	(2)
41.	(4)
42.	(4)
43.	(2)
44.	(1)
45.	(4)
46.	(22)
47.	(8)
48.	(5)
49.	(5)
50.	(6)

MATHEMATICS

51.	(2)
52.	(1)
53.	(4)
54.	(4)
55.	(4)
56.	(3)
57.	(4)
58.	(1)
59.	(2)
60.	(1)
61.	(1)
62.	(3)
63.	(4)
64.	(3)
65.	(4)
66.	(2)
67.	(1)
68.	(4)
69.	(1)
70.	(3)
71.	(0)
72.	(1)
73.	(30)
74.	(2)
75.	(3)

SECTION-I (PHYSICS)

1. **(1)**

Given, $X_C = 400\pi\Omega$, $f = 100\text{Hz}$

At resonance, current is maximum and impendence will be minimum

$\therefore \quad X_L = X_C$

$\Rightarrow \quad \omega L = 400\pi$

$\Rightarrow \quad L = \frac{400\pi}{2\pi \times 100} = 2 \text{ H}$

$i_{\max} = \frac{V}{Z_{\min}}$

$Z_{\min} = R = 8\text{k}\Omega$

$i_{\max} = \frac{500}{8 \times 10^3}$

$i_{\max}$ = 62.5 mA

[26 Aug, 2021 (Shift-I)]

2. **(1)**

$X_C = \frac{1}{\omega C} = 10^4 \Omega$

$\Rightarrow I_{rms} = \frac{E_{rms}}{\sqrt{2} X_C} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20 \text{ mA}$

3. **(3)**

$\Delta x = d \sin\theta = \frac{dy}{D}$

$= \frac{10^{-3} \times 5.24 \times 10^{-3}}{2}$

= 2.62μm

[2 Sep, 2020 (Shift-I)]

4. **(2)**

$U \leq x \leq 1$

For

$K.E. = E_0$

$\lambda = \frac{h}{p} = \frac{5}{\sqrt{2mk.E}}$

$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$

$x > 1$

$K.E. = 2E_0$

$\lambda_2 = \frac{h}{\sqrt{2m(2E_0)}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$

5. **(2)**

$\frac{hc}{\lambda} - \phi = E \qquad \text{….(i)}$

$\frac{hc}{\frac{\lambda}{3}} - \phi = E' \qquad \text{….(ii)}$

Eq. (ii) – eq. (i) gives

$\frac{hc}{\lambda} - \frac{hc}{\frac{\lambda}{3}} = E - E'$

$\Rightarrow \quad -2\frac{hc}{\lambda} = E - E'$

$\Rightarrow \quad E' = E + \frac{2hc}{\lambda}$

$E' = \frac{E\lambda + 2hc}{\lambda}$

[29 July, 2022 (Shift-I)]

6. **(2)**

First minima $= \frac{D\lambda}{a}$

First maxima $= \frac{3D\lambda}{2a}$

$\Rightarrow \frac{3D\lambda}{2a} = \frac{(540)D}{a} \Rightarrow \lambda = 360 \text{ nm}$

7. **(1)**

By working for locus of a point spaced equally from the two point sources we can see that the central fringe will be Circular on P_1 and straight lines on P_2

8. **(4)**

$A_2P - A_1P = \frac{5\lambda}{2}$ (Condition of 3rd minima)

$\Rightarrow \quad \sqrt{D^2 + a^2} - D = \frac{5\lambda}{2}$

$\Rightarrow \quad D\left(1 + \frac{a^2}{D^2}\right)^{1/2} - D = \frac{5\lambda}{2}$

$\Rightarrow \quad \frac{1}{2}\frac{a^2}{D^2} = \frac{5\lambda}{2}$

$\Rightarrow \quad a = \sqrt{5\lambda D}$

$= \sqrt{5 \times 600 \times 10^{-9} \times 0.03}$

= 0.3 mm

[29 Jan, 2023 (Shift-I)]

9. **(4)**

Resonance

$\omega = \frac{1}{\sqrt{\text{LC}}} = \frac{1}{\sqrt{3 \times 10^{-3} \times 4 \times 10^{-3}}}$

$\omega = \frac{10^3}{\sqrt{12}} \text{rad/s}$

10. **(1)**

Resolving power, $R_P \propto \frac{1}{\lambda}$

Limit of resolution $\propto \lambda$

and $\lambda \propto \frac{1}{mv}$

$\therefore \quad LR \propto \frac{1}{mv}$

$\Rightarrow \quad \frac{(LR)_\alpha}{(LR)_e} = \frac{m_e}{m_\alpha} = \frac{m_e}{4m_p} = \frac{1}{4\times 1830}$

$(LR)_\alpha = \frac{1}{7320}(LR)_e$

[18 March, 2021 (Shift-II)]

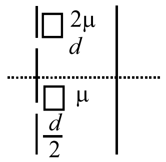
11. **(2)**

since, $\Delta x = \frac{D}{d}(\mu - 1)t$

$\Rightarrow \mu - 1 = \frac{\Delta x d}{Dt} = \frac{5\times 10^{-3}\times 2\times 10^{-3}}{1\times 50\times 10^{-6}} = 0.2$

$\mu = 1 + 0.2 = 1.2$

12. **(2)**



$(2\mu - 1)\frac{dD}{x} - (\mu - 1)\frac{dD}{2x} = \frac{\beta}{2}$

$(4\mu - 2 - \mu + 1)\frac{dD}{2x} = \frac{\lambda D}{2x}$

$(3\mu - 1)d = \lambda$

$d = \frac{\lambda}{3\mu - 1}$

[12 April, 2019 (Shift-I)]

13. **(1)**

$\Rightarrow \quad \sqrt{3x^2 + x^2} - x = \frac{3\lambda}{2}$

$\Rightarrow \quad 2x - x = \frac{3\lambda}{2}$

$\Rightarrow \quad x = 1.5\,\lambda$

[10 Jan, 2019 (Shift-II)]

14. **(2)**

Path difference at central maxima $\Delta x = (\mu - 1)t$, whole pattern will shift by same amount which will be given by $(\mu - 1)t\frac{D}{d} = n\frac{\lambda D}{d}$

$\Rightarrow \quad (\mu - 1)\,t = n\lambda$

$\Rightarrow \quad \mu = \frac{n\lambda}{t} + 1$

$= \frac{5\times 600\times 10^{-9}}{10^{-5}} + 1$

$= 1.3$

[9 April, 2019 (Shift-I)]

15. **(4)**

Power = Energy of photon × Number of photons emitted per sec

$\Rightarrow \quad P = n \times \frac{hc}{\lambda}$

According to question,

$P_x = 3P_\lambda$

$\Rightarrow \quad n_1\frac{hc}{\lambda_1} = 3\times\frac{n_2}{\lambda_2}$

$\Rightarrow \quad \frac{n_1}{n_2} = 3\times\frac{\lambda_1}{\lambda_2}$

$= 3\times\frac{2}{600} = \frac{1}{100}$

[3 Sep, 2020 (Shift-II)]

16. **(3)**

As power factor $\cos\phi$

Where, ϕ is phase difference between v

And i

$\cos\left(\frac{5\pi}{6} - \frac{2\pi}{3}\right) = \cos\frac{\pi}{6} = 0.866$

As current lags voltage

Therefore, 0.866 lags.

17. **(2)**

For maximum kinetic energy, we need to take higher frequency.

i.e. $f = \frac{6\times 10^{15}}{2\pi}Hz$

$K_{max} = hf - \phi$

$= \frac{4\pi}{3}\times 10^{-15}\times\frac{6\times 10^{15}}{2\pi} - 2$

$= 4 - 2$

$= 2eV$

[29 June, 2022 (Shift-II)]

18. **(1)**

$\lambda_e = \frac{h}{p_e}$ .....(i)

and $\lambda_{ph} = \frac{h}{p_{ph}}$ ....(ii)

According to the question,

$\frac{\lambda_e}{\lambda_{ph}} = \frac{3}{2}$

$\Rightarrow \quad \frac{E_e}{E_{ph}} = \frac{\frac{1}{2}p_e v}{p_{ph}c}$ ....(iii)

$= \frac{1}{2}\frac{\lambda_{ph}}{\lambda_e}\frac{v}{c}$

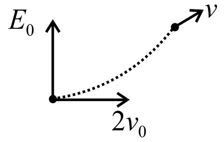
$= \frac{1}{2}\times\frac{2}{3}\frac{v}{c}$

$= \frac{v}{3c}$

[26 June, 2022 (Shift-I)]

19. **(1)**

Given $\vec{v} = 2v_0\hat{i}$ and $\vec{E} = E_0\hat{j}$



$$\vec{v} = 2v_0\hat{i} + \left(\frac{eE_0t}{m}\right)\hat{j}$$

$$[\vec{v}]_{\text{at}\,t=2\text{sec}} = 2v_0\hat{i} + \frac{2eE_0}{m}\hat{j}$$

$$v = |\vec{v}| = 2\sqrt{v_0^2 + \left(\frac{eE_0}{m}\right)^2}$$

de Broglie wavelength is given by $\lambda = \dfrac{h}{mv}$

$$\lambda_{\text{initial}} = \frac{h}{2mv_0}$$

$$\lambda_{\text{final}} = \frac{h}{2m\sqrt{v_0^2 + \left(\frac{eE_0}{m}\right)^2}}$$

$$= \frac{h}{2mv_0}\left(1 + \left(\frac{eE_0}{v_0m}\right)^2\right)^{-1/2}$$

$$= \frac{h}{2mv_0}\left(1 - \frac{1}{2}\left(\frac{eE_0}{v_0m}\right)^2\right)$$

Magnitude of fractional change in de Broglie wavelength is

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2}\left(\frac{eE_0}{mv_0}\right)^2$$

[27 July, 2022 (Shift-I)]

20. **(3)**

From Einstein-Photoelectric effect,

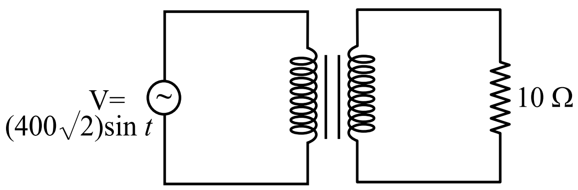
$$\frac{1}{2}\text{mV}^2_{\text{max}} = hf - \phi_0$$

Photoelectric effect can be explained by particle nature of light. Threshold wavelengths is maximum wavelength at which emission takes place.

[27 July, 2022 (Shift-II)]

21. **(50)**

$$\frac{P_s}{P_p} = \frac{v_si_s}{v_pi_p} = 0.8$$



$$\Rightarrow \frac{v_s \cdot v_s/10}{v_p \cdot i_p} = 0.8$$

$$\because \text{v}_\text{s} = \frac{\text{v}_\text{p}}{2}$$

$$\Rightarrow \frac{v_p^2}{40v_pi_p} = 0.8 \Rightarrow i_p = \frac{v_p}{32} = 12.5 \text{ A}$$

22. **(10)**

$\omega = 2\pi f = 100\pi$ rad/s

$X_L = \omega L = 100\,\pi \times 10\pi \times 10^{-3}$

$= 10\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi} \times \frac{1}{\frac{500}{\pi} \times 10^{-6}} = \frac{0.2 \times 10^6}{1 \times 10^4} = 20\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(10\sqrt{3})^2 + (20-10)^2}$$

$$= \sqrt{300+100}$$

$$= \sqrt{400}$$

$= 20\ \Omega$

$$I = \frac{200}{20} = 10\text{A}$$

[25 June, 2022 (Shift-II)]

23. **(50)**

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$I = I_0\cos^2\frac{\phi}{2}$$

$$= 100 \times \cos^2\frac{\pi}{4}$$

$= 50$ Lumens

[6 Sep, 2020 (Shift-II)]

24. **(9)**

Using Malu's law,

$I' = I\cos^2\phi$

$$I' = \frac{I_0}{2} \times \cos^2 53°$$

$$= \frac{I_0}{2} \times \left(\frac{3}{5}\right)^2$$

$$= \frac{9I_0}{50}$$

$x = 9$

[24 Feb, 2021 (Shift-I)]

25. **(11)**

$$hv = \frac{1240}{400}eV$$

$$= 3.1\,ev$$

$$K.E._{\text{max}} = (3.1-2)eV$$

$$= 1.1eV$$

SECTION-II (CHEMISTRY)

26. **(3)**
The correct sequence for the given columns is given as, (I) - (R), (II) - (P), (III) - (S), (IV) - (Q).
[27 July, 2022 (Shift-II)]

27. **(4)**
The ratio of Δ for tetrahedral and octahedral complex is given as $\Delta_t = \frac{4}{9}\Delta_0$. So Δ_t for

$$\left[CoCl_4\right]^{2-} = \frac{4}{9} \times \frac{18000}{0.8}\ cm^{-1} = 10000\ cm^{-1}$$

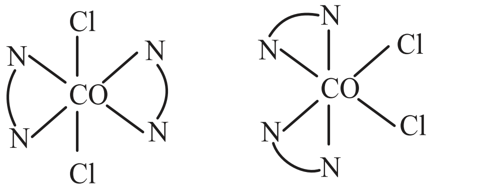
$|CFSE| = 1.2\ \Delta t$

28. **(3)**
Cis–$[CoCl_2(en)_2]^+$ is optically active.

29. **(3)**
F is the most electronegative element in periodic table and hence shows only negative oxidation state i.e. 1.
F can not expand its octet due to unavailability of vacant d-orbital.
[27 June, 2022 (Shift-II)]

30. **(4)**
Amylose contains α–glycosidic linkage only.
[24 June, 2022 (Shift 1)]

31. **(2)**



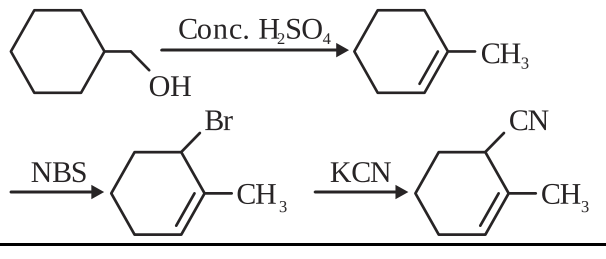
Trans-isomer [optically inactive]

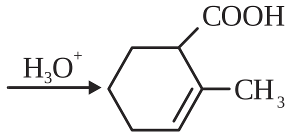
cis-isomer [Optically active]

[11 April, 2023 (Shift-I)]

32. **(3)**
DiBAL-H reduces cyanide, esters, lactones, amide & carboxylic acid into corresponding aldehyde (partial reduction).
Therefore, ester group is reduced into aldehyde.
[26 July, 2022 (Shift-II)]

33. **(3)**





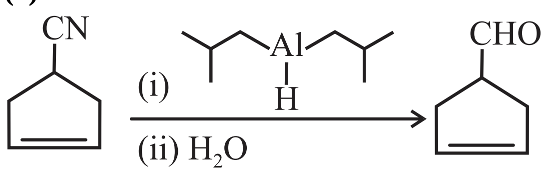
34. **(1)**

$$PhCOOH \xrightarrow{NH_3} \underset{[A]}{PhCOONH_4} \xrightarrow{Heat}$$

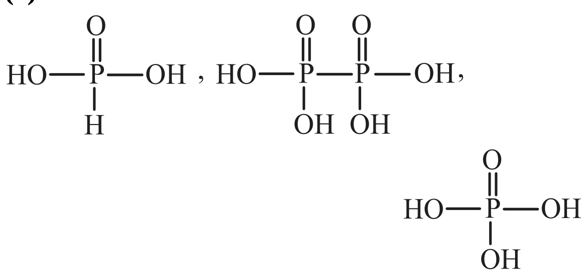
$$\underset{[B]}{PhCONH_2} \xrightarrow{P_2O_5} \underset{[C]}{PhCN} \xrightarrow[Ni/\Delta]{H_2} \underset{[D]}{PhCH_2NH_2}$$

$$PhCONH_2 \xrightarrow[KOH]{Br_2} PhNH_2$$

35. **(4)**



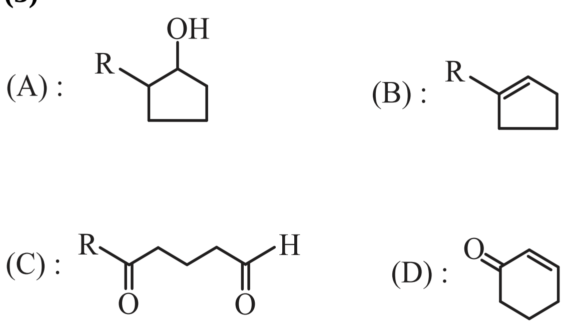
36. **(2)**



[15 April, 2023 (Shift-I)]

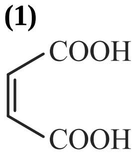
37. **(1)**
$[Fe(CN)_6]^{4-}$ = Octahedral, d^2sp^3, low spin complex. Diamagnetic.
[28 June, 2022 (Shift-I)]

38. **(3)**



[11 April, 2023 (Shift-I)]

39. **(1)**



is most acidic

40. **(2)**

Due to involvement of ℓ.p. of nitrogen in resonance in pyrrole (B), it will show least basic character

: Nitrogen ℓ.p. participate in resonance and increase the stability of the compound due to aromicity.

B < D < C < A

[9 Jan, 2020 (Shift-II)]

41. **(4)**

For iodoform reaction $CH_3-\underset{\underset{OH}{|}}{CH}-$ or $CH_3-\overset{\overset{O}{||}}{C}-$

group should be present.

42. **(4)**

Melting point order Si > Ge > Sn < Pb.

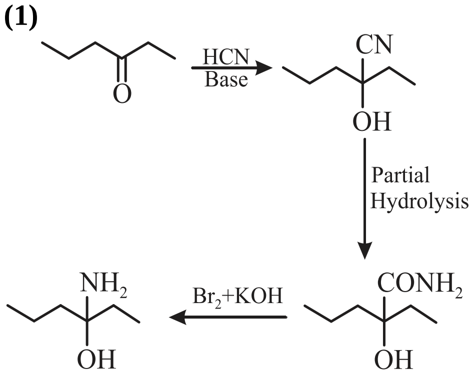
43. **(2)**

The order of the E-H bond's bond dissociation energy in group 15 hydrides is:

$NH_3 > PH_3 > AsH_3 > SbH_3$

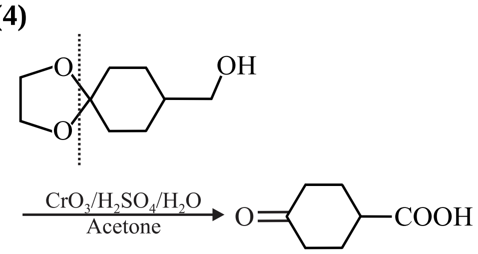
[30 Jan, 2023 (Shift-II)]

44. **(1)**



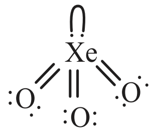
[1 Feb, 2023 (Shift-II)]

45. **(4)**



46. **(22)**

$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$



$XeOF_4$ contains 15 lone-pairs

[18 March, 2021 (Shift-II)]

47. **(8)**

Self and cross-aldol condensation reaction.

$$CH_3-CH_2-\underset{\underset{O}{||}}{C}-H \xrightarrow[\Delta]{OH^{\ominus}} CH_3-CH_2-\underset{\underset{H}{|}}{C}=\underset{\underset{CH_3}{|}}{C}-CHO$$

$$CH_3-\underset{\underset{CH_3}{|}}{CH}-\underset{\underset{O}{||}}{C}-CH_3 \xrightarrow[\Delta]{OH^{\ominus}} \text{2 products (two } \alpha\text{-hydrogens)}$$

Each product shows Geometrical Isomerism.

[29 June, 2022 (Shift-II)]

48. **(5)**

$$2MnO_2 + 4KOH + O_2 \xrightarrow{\text{Heat}} \underset{(A)}{2K_2MnO_4} + 2H_2O$$

$$\underset{(A)}{2K_2MnO_4} + 4HCl \rightarrow \underset{(B)}{2KMnO_4} + MnO_2 + H_2O + KCl$$

$$MnO_4^- + 8H^+ + 5Fe^{2+} + 5e^- \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$$

[29 June, 2022 (Shift-1)]

49. **(5)**

O.S. of sulphur in $H_2S_2O_8$, H_2SO_5, H_2SO_4, $H_2S_2O_7$ and SO_2Cl_2 is equal to +6.

50. **(6)**

$3 \times 2 \times 1 = 6$

[24 Jan, 2023 (Shift-I)]

SECTION-III (MATHEMATICS)

51. **(2)**

Given equation of rearrangement gives

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{y^2 + 16x^2}}{x}$$

Put $y = V \cdot x$ in the above eq^n and

$\frac{dy}{dx} = V + x \cdot \frac{dV}{dx}$, we get

$$\Rightarrow V + x\frac{dV}{dx} = V + \sqrt{V^2 + 16}$$

$$\Rightarrow x\frac{dV}{dx} = \sqrt{V^2 + 16}$$

Apply variable separable method, we get

$$\Rightarrow \int \frac{dV}{\sqrt{V^2 + 16}} = \int \frac{dx}{x} + \ln C$$

$$\Rightarrow \ell n\left|V+\sqrt{V^2+16}\right| = \ell nCx$$

$$\Rightarrow \frac{y}{x}+\frac{\sqrt{y^2+16x^2}}{x}=Cx$$

Given $y\,(1) = 0 \Rightarrow C = 4$

Now at $x = 2$

$$\Rightarrow \frac{y}{2}+\frac{\sqrt{y^2+16\cdot 4}}{2}=4\cdot 2$$

$$\Rightarrow y+\sqrt{y^2+64}=16$$

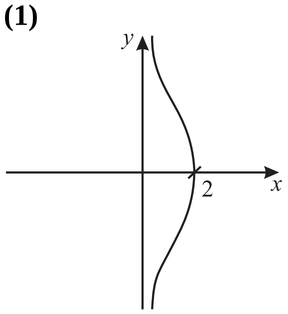
$$\Rightarrow y^2+64=(16-y)^2$$

$$=y^2+64=y^2-32y+16^2$$

$$\Rightarrow y=6$$

[29 June, 2022 (shift-I)]

52. **(1)**



$$\text{Area } = \int_{-\infty}^{\infty} x\,dy$$

$$=\int_{-\infty}^{\infty}\frac{2}{1+y^2}dy$$

$= 2\pi$

53. **(4)**

Let $\bar{r}=x\hat{i}+y\hat{i}+z\hat{k}$

$$\bar{r}\times\left(\hat{i}+2\hat{j}-\hat{k}\right)=\hat{i}+\hat{k}$$

$$-(y+2z)\hat{i}+\left(x+z\right)\hat{j}+\left(2x-y\right)\hat{k}=\hat{i}+\hat{k}$$

Compare

$\bar{r}=x\hat{i}+\left(2x-1\right)\hat{j}-x\hat{k}$ for $x\in R$

54. **(4)**

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)=\ln x\Rightarrow x\frac{dy}{dx}=x\ln x-x+c\Rightarrow c=0$$

(A (1, 1) and $\frac{dy}{dx}=-1$)

$$x\frac{dy}{dx}=x\ln x-x\Rightarrow dy=\left(\ln x-1\right)dx$$

$$\Rightarrow y=x\ln x-x-x+c_1\Rightarrow c_1=3$$

55. **(4)**

Given $\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$

$\vec{a}\times(\vec{c}-\vec{b})=0$

$\lambda\vec{a}=\vec{c}-\vec{b}$

$$\Rightarrow\vec{c}=\vec{b}+\lambda\vec{a}$$

$$\Rightarrow\vec{c}=\left(p\hat{i}+13\hat{j}+4\hat{k}\right)+\lambda\left(4\hat{i}+3\hat{j}+10\hat{k}\right)$$

$$=\left(p+4\lambda\right)\hat{i}+\left(13+3\lambda\right)\hat{j}+\left(4+10\lambda\right)\hat{k}$$

….(1)

Given $\vec{a}\cdot\vec{c}=75$

$\Rightarrow 4p+125\lambda=-4$ …. (2)

$\vec{c}\cdot\left(\hat{i}+2\hat{j}-3\hat{k}\right)=4$

$\Rightarrow p-20\lambda=-10$ ….. (3)

Solving 2 & 3

$$\lambda=\frac{36}{205},\ p=-\frac{266}{41}$$

[8 April, 2023 (shift-I)]

56. **(3)**

$$\int_1^2\frac{9x+4}{x^5+3x^2+x}dx$$

$$=\int_1^2\frac{9x^{-4}+4x^{-5}}{1+3x^{-3}+x^{-4}}dx$$

Put $1+3x^{-3}+x^{-4}=t\Rightarrow\left(-9x^{-4}-4x^{-5}\right)dx=dt$

$$=-\int\frac{1}{t}dt=\ln\left(\frac{1}{t}\right)$$

$$=\left[\ln\left(\frac{1}{1+3x^{-3}+x^{-4}}\right)\right]_1^2=\ln\left(\frac{80}{23}\right).$$

57. **(4)**

Rearranging the given equation we get

$$\Rightarrow\frac{dy}{dx}=-\frac{2^x\left(2^y-1\right)}{2^y\left(2^x-1\right)}$$

$$\Rightarrow\int\frac{2^y}{2^y-1}dy=-\int\frac{2^x}{2^x-1}dx$$

$$\Rightarrow\frac{\log_e\left(2^y-1\right)}{\log_e 2}=-\frac{\log_e\left(2^x-1\right)}{\log_e 2}+\frac{\log_e c}{\log_e 2}$$

$$\Rightarrow\left|\left(2^y-1\right)\left(2^x-1\right)\right|=c$$

$\because y(1)=1$

$\therefore c=1$

$$\Rightarrow\left(2^y-1\right)\left(2^x-1\right)=1$$

For $x=2$

$$\left(2^y-1\right)3=1$$

$$\Rightarrow 2^y-1=\frac{1}{3}\Rightarrow 2^y=\frac{4}{3}$$

Taking log to base 2.

$$\therefore y=2-\log_2 3$$

[27 June, 2022 (Shift-I)]

58. **(1)**

Here, $a^{3\log_a 2x}=8x^3$ similarly for other terms.

$$f(x)=\int\frac{8x^3+20x^2}{x^4+5x^3-7x^2}dx=4\int\frac{2x+5}{x^2+5x-7}dx$$

$$=4\ln\left|x^2+5x-7\right|+c$$

$f(1) = 4\ln 1 + c \Rightarrow c = 0$

$f(0) = 4 \ln 7$

[25 Feb, 2021 (Shift-II)]

59. **(2)**

$$I=\int_5^{37}\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}.dx$$

$$=\int_5^{37}\sqrt{\left(\sqrt{x-1}-2\right)^2}+\sqrt{\left(\sqrt{x-1}-3\right)^2}.dx$$

$$=\int_5^{37}\left(\sqrt{x-1}-2\right)dx+\int_5^{10}\left(3-\sqrt{x-1}\right)dx+\int_{10}^{37}\left(\sqrt{x-1}-3\right)dx$$

= 122.

60. **(1)**

Given $\vec{a}=\alpha\hat{i}+\hat{j}-\hat{k}$ & $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 1 & -1\\ 2 & 1 & -\alpha\end{vmatrix}$$

$$=(1-\alpha)\hat{i}+\left(\alpha^2-2\right)\hat{j}+(\alpha-2)\hat{k}$$

Projection of $\vec{b}\times\vec{a}$ on $\hat{i}+\hat{j}+\hat{k}$

$$\Rightarrow\frac{(\vec{b}\times\vec{a})\cdot(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}=\sqrt{3}$$

$$\left[\because \text{ projection of } \vec{a} \text{ on } \vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right]$$

$$\Rightarrow\frac{-\alpha^2+3}{\sqrt{3}}=\sqrt{3}\Rightarrow\alpha=0$$

[26 July, 2022 (Shift-I)]

61. **(1)**

Given data

$$\left(1+x^2y^2\right)dx=ydx+xdy,\ y(1)=1$$

$$y(2)=\alpha$$

$$dx=\frac{d(xy)}{1+(xy)^2}\Rightarrow\int dx=\int\frac{d(xy)}{1+(xy)^2}$$

$$x=\tan^{-1}\left(xy\right)+c$$

$x = 1$ then $y = 1$

$$\Rightarrow c=1-\frac{\pi}{4}$$

$$\Rightarrow x=\tan^{-1}(xy)+1-\frac{\pi}{4}$$

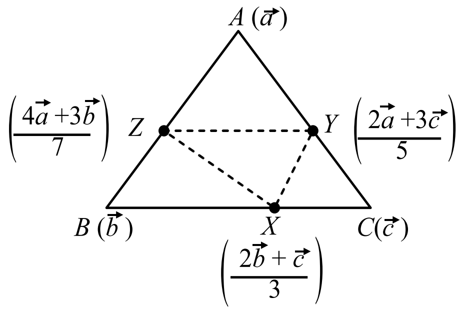
$$x=2, y=\alpha$$

$$2=\tan^{-1}2\alpha+1-\frac{\pi}{4}$$

$$\Rightarrow\tan^{-1}2\alpha=1+\frac{\pi}{4}$$

[11 April, 2023 (Shift-I)]

62. **(3)**



$$2\Delta ABC=\left|\overrightarrow{BC}\times\overrightarrow{BA}\right|=\left|\left(\vec{c}-\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|$$

$$=\left|\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)\right|$$

$$2\Delta XYZ=\left[\left(\frac{2\vec{a}+3\vec{c}}{5}\right)-\left(\frac{2\vec{b}+\vec{c}}{3}\right)\right]\times\left[\left(\frac{4\vec{a}+3\vec{b}}{7}\right)-\left(\frac{2\vec{b}+\vec{c}}{3}\right)\right]$$

$$=\frac{\left(2\vec{a}+3\vec{c}\right)\times\left(4\vec{a}+3\vec{b}\right)}{35}-\frac{\left(2\vec{a}+3\vec{c}\right)\times\left(2\vec{b}+\vec{c}\right)}{15}$$

$$-\frac{\left(2\vec{b}+\vec{c}\right)\times\left(4\vec{a}+3\vec{b}\right)}{21}+0$$

$$=\left(\frac{6\vec{a}\times\vec{b}+12\vec{c}\times\vec{a}+9\vec{c}\times\vec{b}}{7\times5\times3}\right)3$$

$$-\left(\frac{4\vec{a}\times\vec{b}+2\vec{a}\times\vec{c}+6\vec{c}\times\vec{b}}{7\times5\times3}\right)\times7$$

$$-\left(\frac{8\vec{b}\times\vec{a}+4\vec{c}\times\vec{a}+3\vec{c}\times\vec{b}}{7\times5\times3}\right)\times5$$

$$=\frac{30\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)}{105}$$

$$=\frac{2\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)}{7}$$

$$\therefore\frac{\Delta ABC}{\Delta XYZ}=\frac{7}{2}$$

[24 Jan, 2023 (Shift-I)]

63. **(4)**

$x = e^{\ln x}$

$$\int\left(e^{(\ln x-1)3x}+e^{(1-\ln x)3x}\right)\ln x dx$$

$$=\int\left(e^{3(x\ln x-x)}+e^{3(x-x\ln x)}\right)\ln x\,dx$$

$$=\int\left(e^{3t}+e^{-3t}\right)dt \qquad \begin{aligned}x\ln x-x&=t\\ \ln x\,dx&=dt\end{aligned}$$

$$=\frac{e^{3t}}{3}+\frac{e^{-3t}}{-3}+\lambda$$

$$=\frac{1}{3}e^{3(x\ln x-x)}-\frac{1}{3}e^{3(x-x\ln x)}+\lambda$$

$$=\frac{1}{3}\left(\frac{x}{e}\right)^{3x}-\frac{1}{3}\left(\frac{e}{x}\right)^{3x}+\lambda$$

[10 April, 2023 (Shift-II)]

64. **(3)**

The given equation can be rewritten as

$$\frac{dy}{dx}+\frac{x^2-1}{x\left(x^2+1\right)}y=\frac{x^2\ln x}{\left(x^2+1\right)} \qquad \ldots\ldots \ (1)$$

Which is linear. Also

$$P=\frac{x^2-1}{x\left(x^2+1\right)} \text{ and } Q=\frac{x^2\ln x}{\left(x^2+1\right)}$$

$$\int P\,dx=\int\left[\frac{2x}{x^2+1}-\frac{1}{x}\right]dx$$

[Resolving into partial fractions]

$$=\ln\left(x^2+1\right)-\ln x$$

$$\therefore \text{I.F.}=e^{\ln\left[\left(x^2+1\right)/x\right]}=\frac{x^2+1}{x}$$

Hence the required solution of equation (1) is

$$\frac{y\left(x^2+1\right)}{x}=\int\frac{\left(x^2+1\right)}{x}\frac{x^2\ln x}{\left(x^2+1\right)}dx+c$$

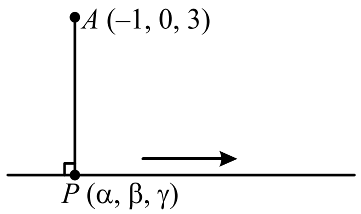
$$=\int x\ln x\,dx+c$$

$$=\frac{1}{2}x^2\ln x-\int\frac{1}{x}\frac{x^2}{2}dx+c$$

$$\therefore y\left(\frac{x^2+1}{x}\right)=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+c$$

65. **(4)**

$$L:\frac{x+1}{-2}=\frac{y}{3}=\frac{z+1}{-1}=\lambda \text{ (let)}$$



Let foot of perpendicular is

$P(-2\lambda-1, 3\lambda, -\lambda-1)$

$$\overrightarrow{PA}=(-2\lambda)\hat{i}+(3\lambda)\hat{j}+(-4-\lambda)\hat{k}$$

Now, $\Rightarrow \overrightarrow{PA}\cdot\vec{b}=0$

$$\Rightarrow \lambda=\frac{-2}{7}$$

$$\Rightarrow P\equiv\left(\frac{-3}{7},\frac{-6}{7},\frac{-5}{7}\right)$$

$$\frac{\beta}{\alpha}=2$$

$$\alpha+\beta+\gamma=-2$$

$$\frac{\gamma}{\beta}=\frac{5}{6}$$

$$\frac{\gamma}{\alpha}=\frac{5}{3}$$

[25 Jan, 2023 (Shift-II)]

66. **(2)**

$$A(x)=\int_0^x y(x)dx=\left(\frac{y}{x}\right)^2$$

Differentiate both sides

$$\Rightarrow y=\frac{x^2\cdot 2yy'-y^2\cdot 2x}{x^4}$$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=\frac{x^2}{2}$$

$$I.F=\frac{1}{x}$$

$$\Rightarrow y\cdot\frac{1}{x}=\int\frac{x}{2}dx+\lambda$$

$$\Rightarrow \frac{y}{x}=\frac{1}{2}\cdot\frac{x^2}{2}+\lambda$$

$$\Rightarrow y=\frac{x^3}{4}+\lambda x$$

$$x=2, y=4$$

$$\Rightarrow \lambda=1$$

$$y=\frac{1}{4}x^3+x.$$

This passes through $\left(\alpha,\frac{5}{4}\right)$

$$\Rightarrow \alpha^3+4\alpha-5=0$$

$$\Rightarrow (\alpha-1)\left(\alpha^2+\alpha+5\right)=0$$

$$\Rightarrow \alpha=1$$

[26 July, 2022 (Shift-I)]

67. **(1)**

$$I=\int_0^\infty\frac{1}{1+e^x}dx+\int_0^\infty\frac{1}{e^x-2}dx+\int_0^\infty\frac{1}{e^x-3}dx$$

$$I=\int_0^\infty\frac{e^{-x}}{1+e^{-x}}dx+\int_0^\infty\frac{e^{-x}}{1-2e^{-x}}dx+\int_0^\infty\frac{e^{-x}}{1-3e^{-x}}dx$$

$$=\int_0^\infty-\frac{\left(-e^{-x}\right)}{1+e^{-x}}dx+\frac{1}{2}\int_0^\infty\frac{(-2)\left(-e^{-x}\right)}{1-2e^{-x}}dx+\frac{1}{3}\int_0^\infty\frac{(-3)\left(-e^{-x}\right)}{1-3e^{-x}}dx$$

$$=\left[-\ln\left(1+e^{-x}\right)\right]_0^\infty+\frac{1}{2}\left[\ln\left|1-2e^{-x}\right|\right]_0^\infty+\frac{1}{3}\left[\ln\left|1-3e^{-x}\right|\right]_0^\infty$$

$$(0+\ln 2)+\frac{1}{2}(0-0)+\frac{1}{3}(0-\ln 2)$$

$$=\frac{2}{3}\ln 2$$

[13 April, 2023(Shift-I)]

68. **(4)**

Shortest distance between the lines

$$=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}=\frac{16}{\sqrt{52}}$$

[24 Jan, 2023 (Shift-I)]

69. **(1)**

Equation of tangent : $y-1=3(x-1)$

i.e. $y=3x-2$ …(1)

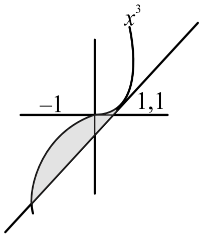
$y=x^3$ …(2)

Solving (1) & (2)

$$(x-1)^2(x+2)=0$$

$$\text{Area} = \int_{-2}^{1}\left(x^3-(3x-2)\right)dx$$

$$=\left[\frac{x^4}{4}-\frac{3x^2}{2}+2x\right]_{-2}^{1}=\frac{27}{4}$$



[12 April, 2023 (Shift-I)]

70. **(3)**

$$\left(xy^3-x^2\right)dy-\left(xy+y^4\right)dx=0$$

$$\Rightarrow y^3(x\,dy-y\,dx)-x(x\,dy+y\,dx)=0$$

$$\Rightarrow x^2y^3\frac{(x\,dy-y\,dx)}{x^2}-x(x\,dy+y\,dx)=0$$

$$\Rightarrow x^2y^3\,d\left(\frac{y}{x}\right)-x\,d(xy)=0$$

$\Rightarrow$ Dividing by x^3y^2, we get

$$\Rightarrow \frac{y}{x}d\left(\frac{y}{x}\right)-\frac{d(xy)}{x^2y^2}=0$$

Now integrating $\frac{1}{2}\left(\frac{y}{x}\right)^2+\frac{1}{xy}=c$

It passes through the point $(4,-2)$

$$\Rightarrow \frac{1}{8}-\frac{1}{8}=c\Rightarrow c=0$$

$$\therefore y^3=-2x$$

71. **(0)**

$$\int_0^{\pi/4}e^{-x}\tan^{25}x\,dx$$

$$\left[-e^{-x}(\tan x)^{25}\right]_0^{\pi/4}+\int_0^{\pi/4}e^{-x}(25)(\tan x)^{24}\sec^2 x\,dx$$

$$=-e^{-\pi/4}+0+25\left[\int_0^{\pi/4}e^{-x}(\tan x)^{26}dx+\int_0^{\pi/4}e^{-x}(\tan x)^{24}dx\right]$$

$$=-e^{-\pi/4}+25\left(\int_0^{\pi/4}e^{-x}\left((\tan x)^{26}+(\tan x)^{24}\right)dx\right)$$

$$\Rightarrow 25\int_0^{\pi/4}e^{-x}\left(\tan^{24}x+\tan^{26}x\right)dx$$

$$-\int_0^{\pi/4}e^{-x}\tan^{25}x\,dx=e^{-\pi/4}$$

[13 April, 2023 (Shift-II)]

72. **(1)**

$$y^2dx+xy\,dy+\left(x^2y^2\right)dy=0$$

$$\frac{y\,dx+x\,dy}{x^2\cdot y^2}+\frac{dy}{y}=0$$

$$\Rightarrow d\left(-\frac{1}{xy}\right)+d(\ln y)=0$$

$$-\frac{1}{xy}+\ln y+c=0$$

Curve passes through (1, 1) $\Rightarrow c=1$

$\therefore\quad 1+\ln x=\frac{1}{x^2}\Rightarrow$ 1 solution

[25 June, 2022 (Shift-I)]

73. **(30)**

$$I_1+I_2=\int_0^{\pi}f(x).\sin x\,dx+\int_0^{\pi}f''(x)\sin x\,dx$$

$$=f(x).-\cos x\Big|_0^{\pi}+\int_0^{\pi}\cos x.f'(x)dx$$

$$+\sin x\,f'(x)\Big|_0^{\pi}-\int_0^{\pi}\cos x.f'(x)dx$$

$$\Rightarrow\quad f(\pi)+f(0)=5(given)$$

$$\Rightarrow\quad f(0)=5-f(\pi)=5-2=3$$

74. **(2)**

$$\int_0^x tf(t)\,dt = x^2 f(x), xf(x) = 2xf(x) + x^2 f'(x)$$

Let $y = f(x), xy = 2xy + x^2 \frac{dy}{dx}, x^2 \frac{dy}{dx} = -xy, \frac{dy}{dx}$

$= -\frac{y}{x}$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}, \log y = -\log x + \log c$$

$$\log xy = \log c, xy = c, c = 6, xy = 6, f(x) = \frac{6}{x}$$

$f(6) = 1$, $f(8) = \frac{6}{8} = \frac{3}{4} = 0.75$, $f(10) =$

$= \frac{6}{10} = \frac{3}{5} = 0.6$

$f(6) + f(8) + f(10) = 1 + 0.75 + 0.60 = 2.35$

75. **(3)**

Any point on the line is

$P = (6r_1 + 2, 3r_1 + 3, -4r_1 - 4)$.

Direction ratios of the line segment PQ are

$\ll 6r_1 + 3, 3r_1 + 1, -4r - 10 \gg$ (where $Q(-1, 2, 6)$),

If 'P' be the foot of altitude drawn from Q to the given line, then

$6(6r_1 + 3) + 3(3r_1 + 1) + 4(4r_1 + 10) = 0$.

$\Rightarrow \quad r_1 = -1$.

Thus, $P = (-4, 0, 0)$

$\therefore$ Required distance $|\overline{PQ}| = \sqrt{9+4+36} = b$

$b = 7$ units., $\alpha = -4$

$\alpha + b = 3$

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