MIND MAP FOR JEE ASPIRANTS

Physics

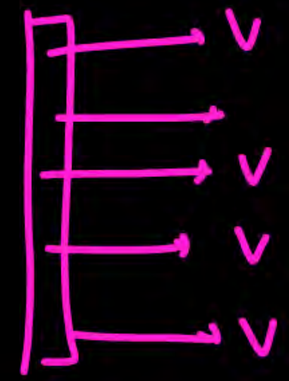
Center of Mass

COM

By- Saleem Ahmed Sir

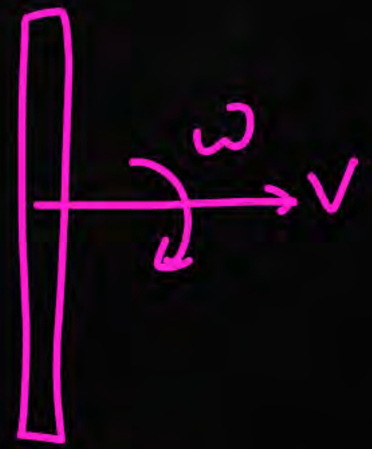
Today's Targets

1. Com
2. [redacted]
3. [redacted]
4. [redacted]



pure translation

fix Axis

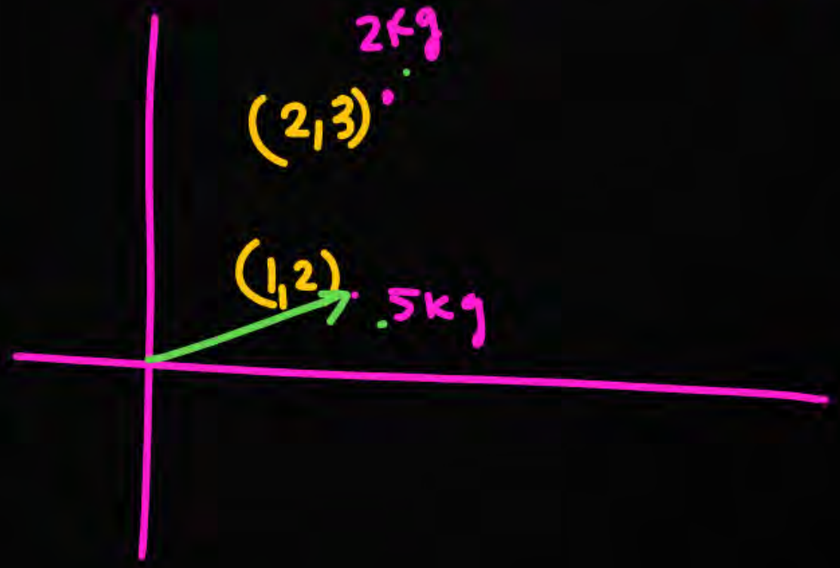


CRTM

$$x_{com} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$x_{com} = \frac{5\times1 + 2\times2 + 4\times4}{}$$

$y_{com} =$



$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{5(\hat{i}+2\hat{j}) + 2(2\hat{i}+3\hat{j}) + 4(4\hat{i}+6\hat{j})}{}$$

Q

$\lambda = \lambda_0 x$

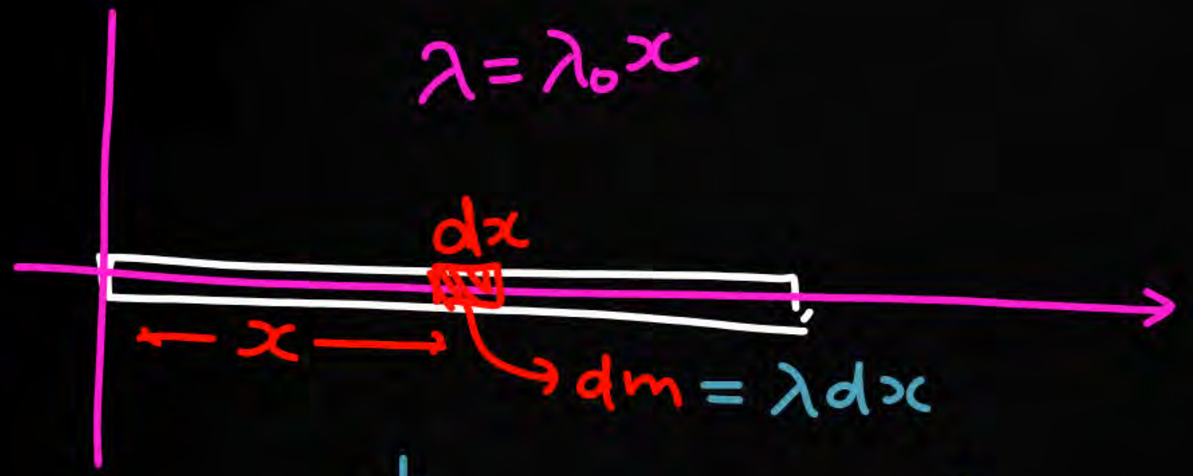
$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$\lambda = \frac{dm}{dx}$$

$$\sigma = \frac{dm}{dA}$$

$$\rho = \frac{dm}{dV}$$

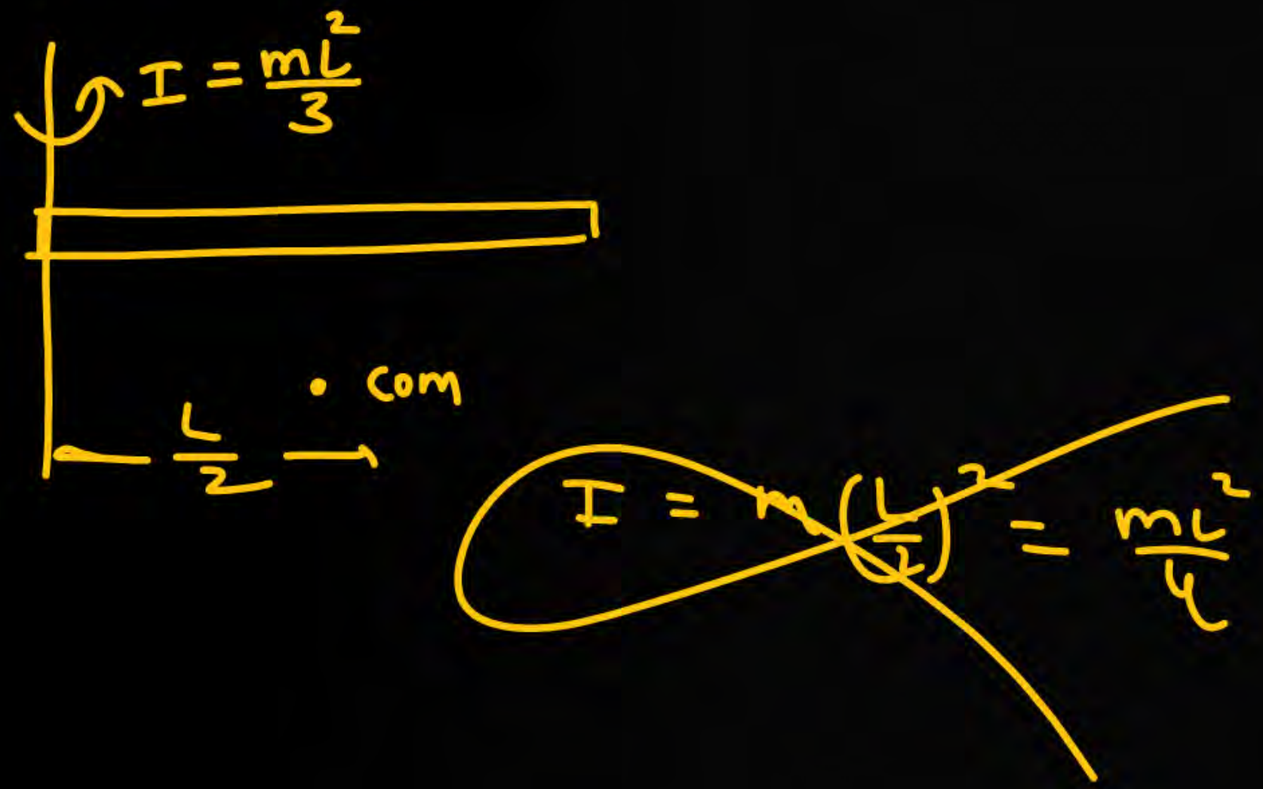
$$x_{cm} = \frac{\int x \cdot dm}{m_{total}}$$



$$x_{cm} = \frac{\int_0^L x \cdot \lambda dx}{\int_0^L \lambda dx} = \frac{\int_0^L x\, \lambda_0 x\, dx}{\int_0^L \lambda_0 x\, dx}$$

Total KE $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$+ \ldots\ldots \infty$


$I = \frac{mL^2}{3}$
COM
$\frac{L}{2}$
$I = m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$

Location of com

com

Ring, Disc, Sphere

com

com

com

triangular plate

$\frac{h}{3}$

(*) Semi Ring

com

$\frac{2R}{\pi}$

(*) Semi Disc

com

$\frac{4R}{3\pi}$

Solid hemisphere

$3R/8$

hollow hemisphere

$R/2$

hollow cone

com

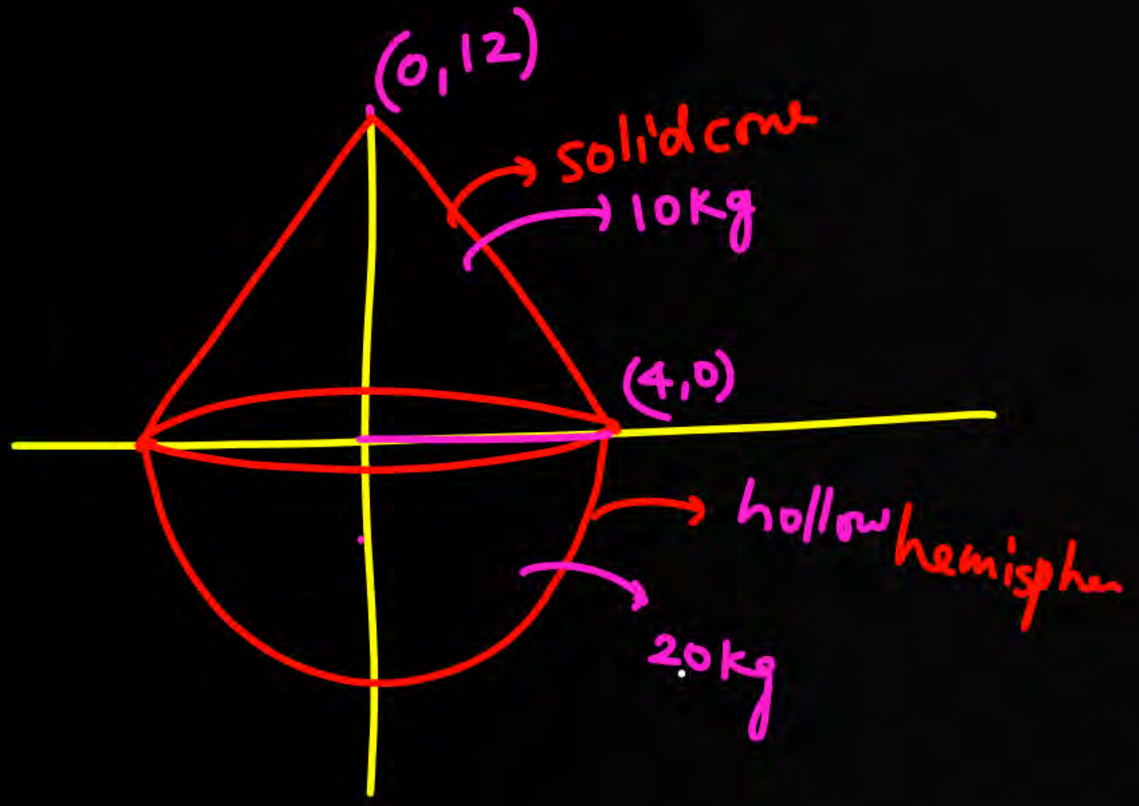
$\frac{h}{3}$

Solid cone

com

$\frac{h}{4}$

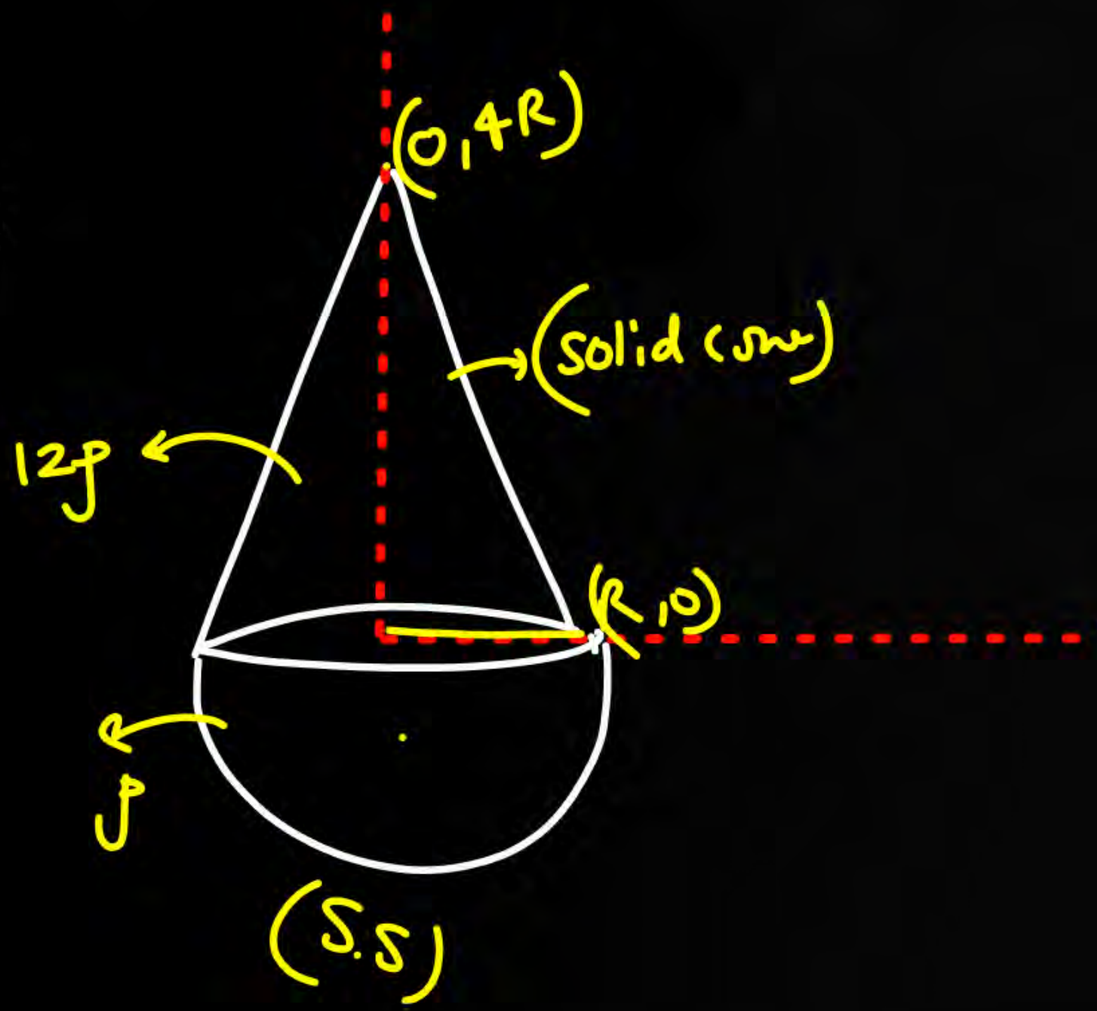
Q



$$x_{cm} = 0$$

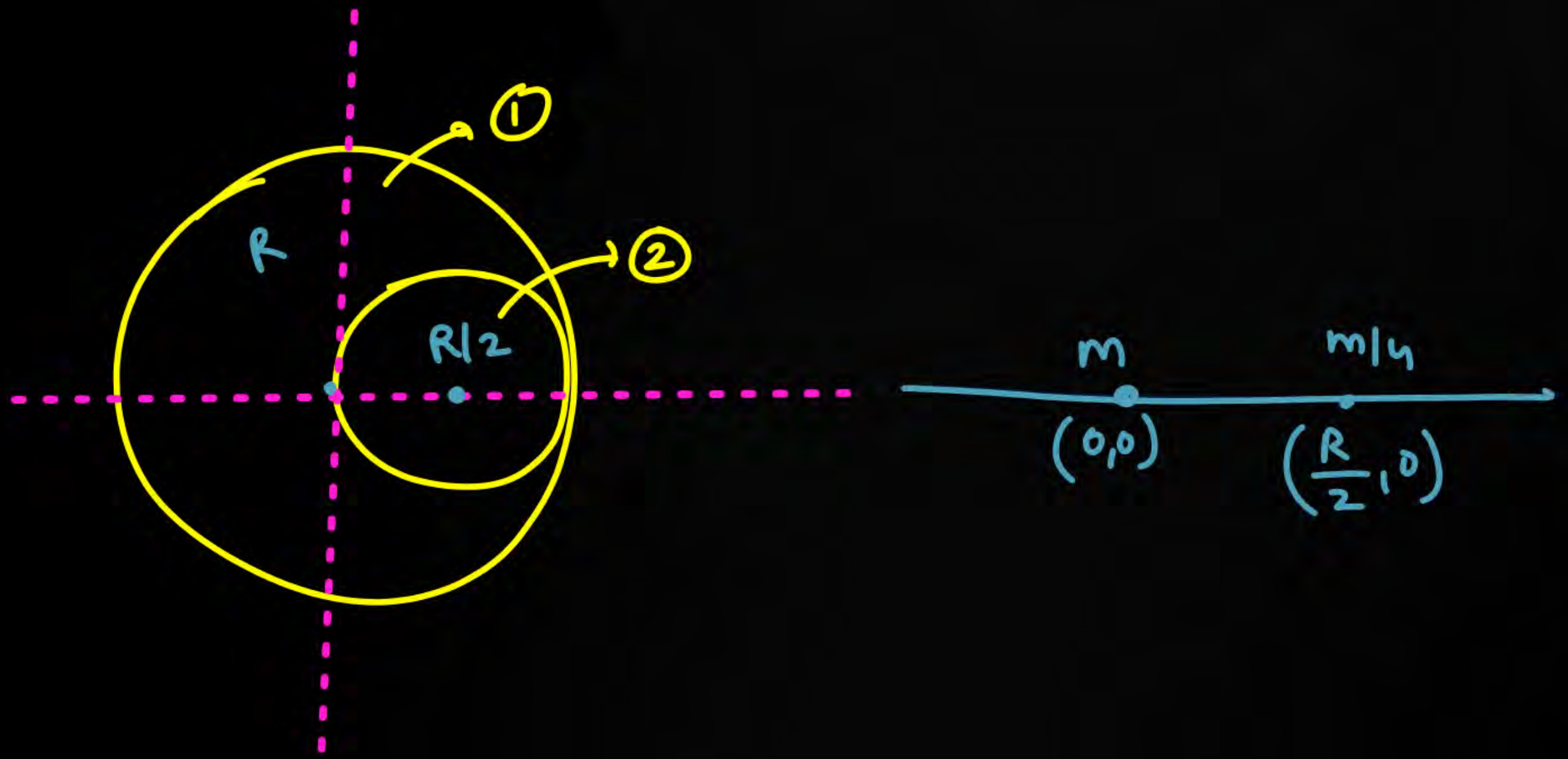
$$y_{cm} = \frac{10 \times 3 + 20(-2)}{10 + 20}$$

Q



$m_1 = 12\rho \times \left(\frac{1}{3}\pi R^2 h\right)$

$m_2 = \rho \frac{2}{3}\pi R^3$


1
2
R
R/2
m
m/4
(0,0)
(R/2,0)

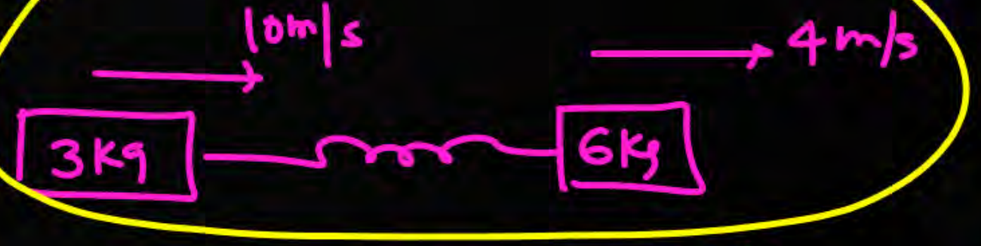
$$\vec{d}_{com} = \frac{m_1\vec{d_1} + m_2\vec{d_2} + \cdots}{m_1 + m_2 + \cdots}$$

$$\vec{v}_{com} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + \cdots}{m_1 + m_2 + \cdots} = \frac{\vec{P}_{net}}{M_{total}}$$

$$\vec{a}_{com} = \frac{m_1\vec{a_1} + m_2\vec{a_2} + \cdots}{m_1 + m_2 + \cdots}$$

$$\vec{a}_{com} = \frac{\vec{F}_{net}}{M_{total}}$$

$P_{net} = (3+6) \times 6$



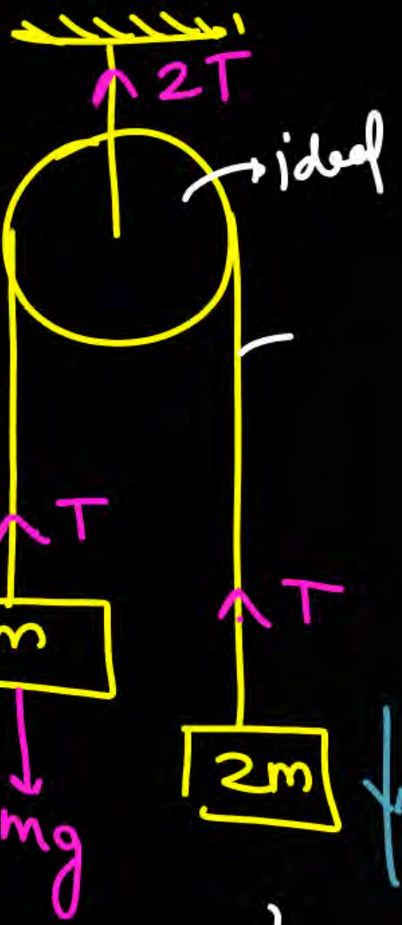
$$v_{com} = \frac{3\times10 + 6\times4}{3+6} = \checkmark$$

$$\vec{v}_{com} = \frac{\vec{P}_{syst\ net}}{M_{total}}$$

$$\vec{P}_{net} = M_{total}\,\vec{v}_{com}$$

$$(F_{net})_{ext} = M_{total}\,\vec{a}_{com}$$

Q



$a = g/3$ ↑

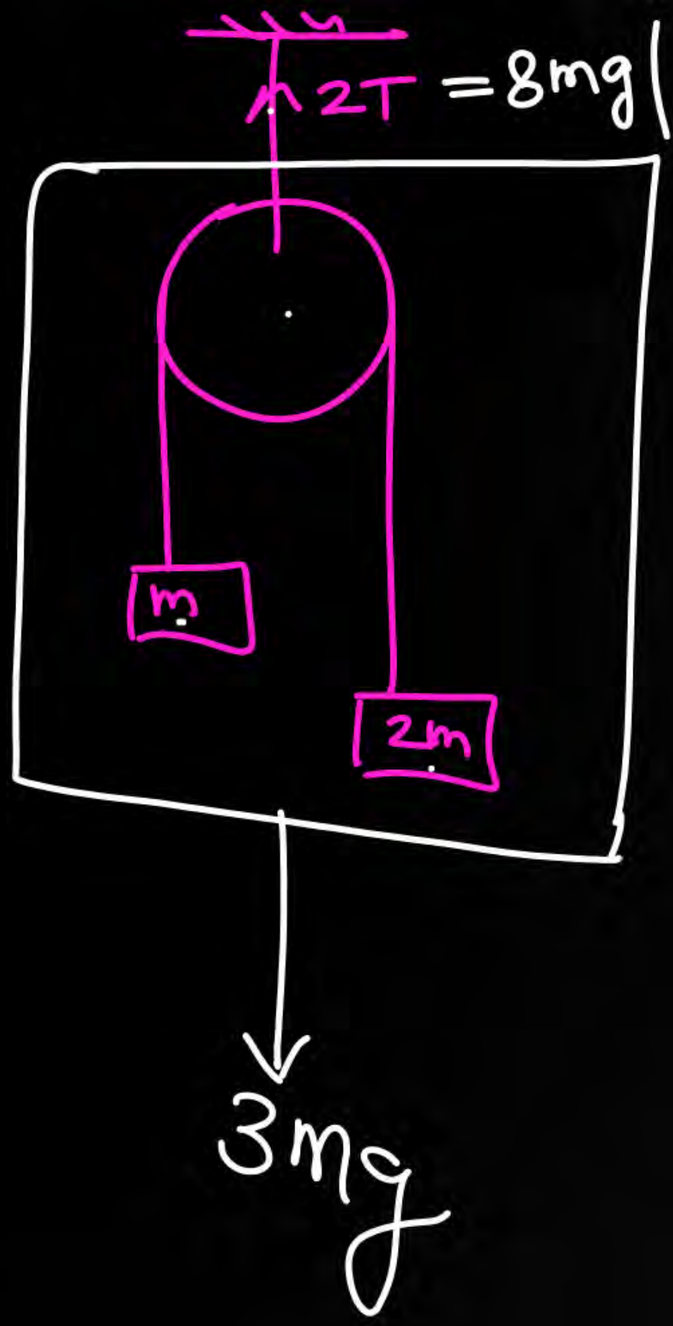
$a = \frac{2mg - mg}{3m} = g/3$ ↓

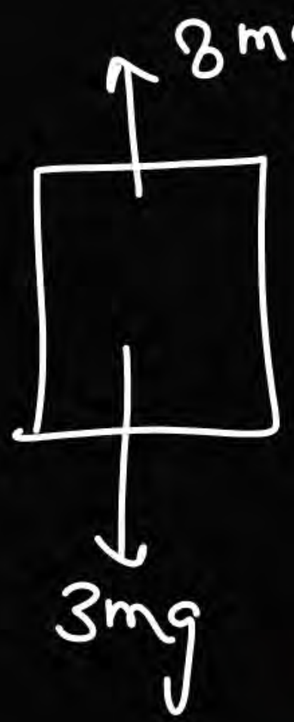
$T - mg = ma$

$T = mg + \frac{mg}{3} = \frac{4mg}{3}$

$a_{com} = \frac{m\,g/3 + 2m.(-g/3)}{m+2m}$

$a_{com} = \frac{-mg}{3 \times 3m} = \frac{-g}{9}$ नीचे





नीचे

$$F_{net} = 3mg - \frac{8mg}{3}$$

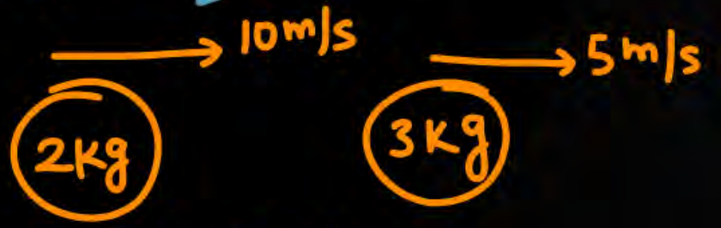
$$F_{net} = \frac{mg}{3} = m_{tot}\, a_{com}$$

$$\frac{mg}{3} = 3m \times a_{com}$$

$$a_{com} = g/9$$

नीचे

$(KE)_i = 100 + \frac{75}{2} = \frac{275}{2}$



$e = 1$

$\equiv$

$e = 1 \Rightarrow \quad P_i = P_f$

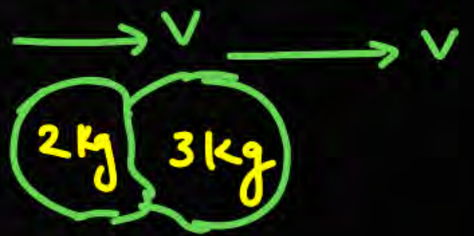
$2 \times 10 + 3 \times 5 = 2v_1 + 3v_2$ ——— (1)

$e = 1 = \frac{v_2 - v_1}{5}$ ——— (2)

$v_1 = 4 \text{ m/s}$

$v_2 = 9 \text{ m/s}$

$(P.E)_{max} = ?$ $\quad (KE)_{min}$



$P_i = P_f$

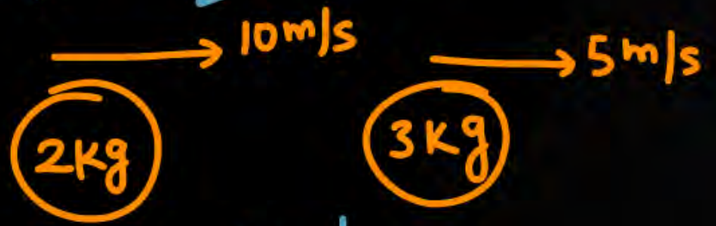
$20 + 15 = 2v + 3v$

$\boxed{v = 7}$

$(KE) = \frac{1}{2} \times 2 \times 7^2 + \frac{1}{2} 3 \times 7^2 = \frac{245}{2}$

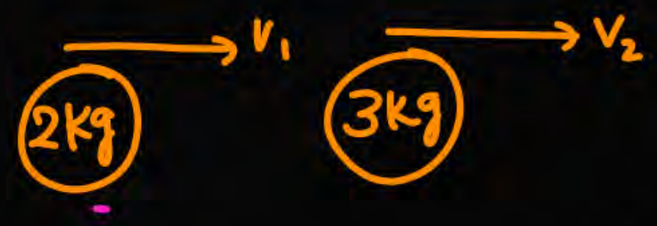
$(PE)_{min} = 15$

$(KE)_i = 100 + \frac{75}{2} = \frac{275}{2}$



$e = \frac{1}{2}$

$\equiv$



$e = 1 \Rightarrow P_i = P_f$

$2\times10 + 3\times5 = 2v_1 + 3v_2$ ——— (1)

$e = \frac{1}{2} = \frac{v_2 - v_1}{5}$ ——— (2)

$(\Delta KE)_{loss} = \frac{1}{2}\mu u_{rel}^2 (1 - e^2)$, $\mu = \frac{m_1 m_2}{m_1 + m_2}$

(3) $e = 0$

loss ke $= \frac{1}{2}\times\frac{6}{5}\times25\,(1-0) = 15$

$20 + 15 = 2v_1 + 3v_2$

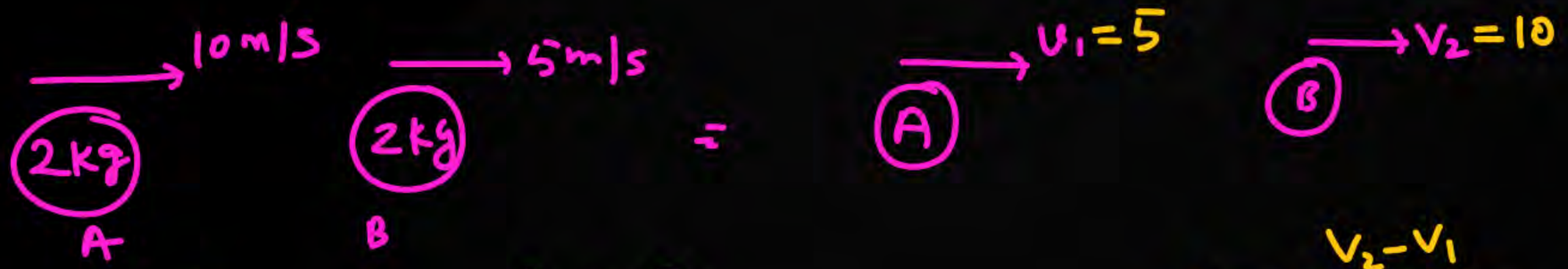
$e = 0 = \frac{v_2 - v_1}{5}$

$v_2 = v_1 = v_0$

$v_0 = 7$

$\therefore$

Q



$e = 1$

$P_i = P_f$

$$20 + 10 = 2u_1 + 2u_2$$

$$u_1 + u_2 = 15$$

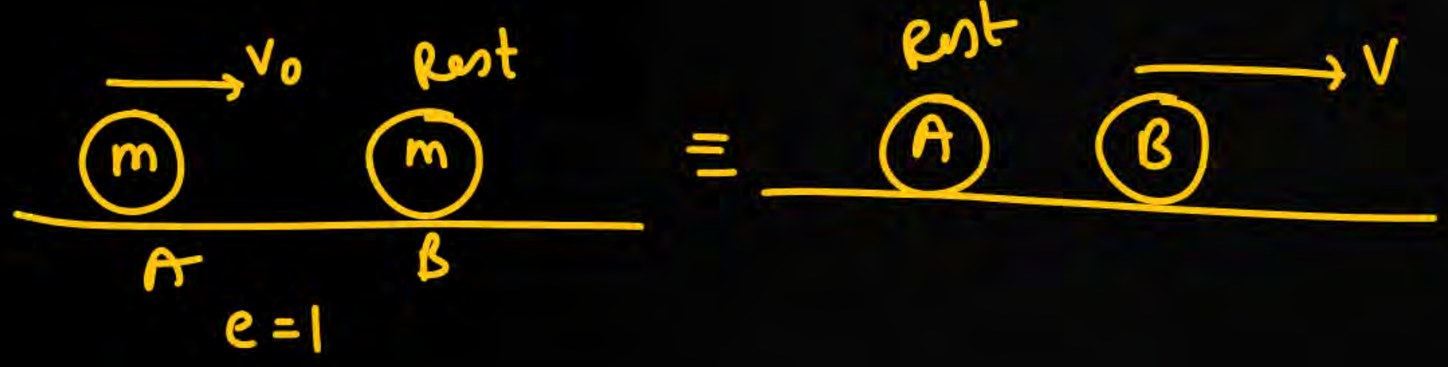
$$v_2 - v_1 = 5$$

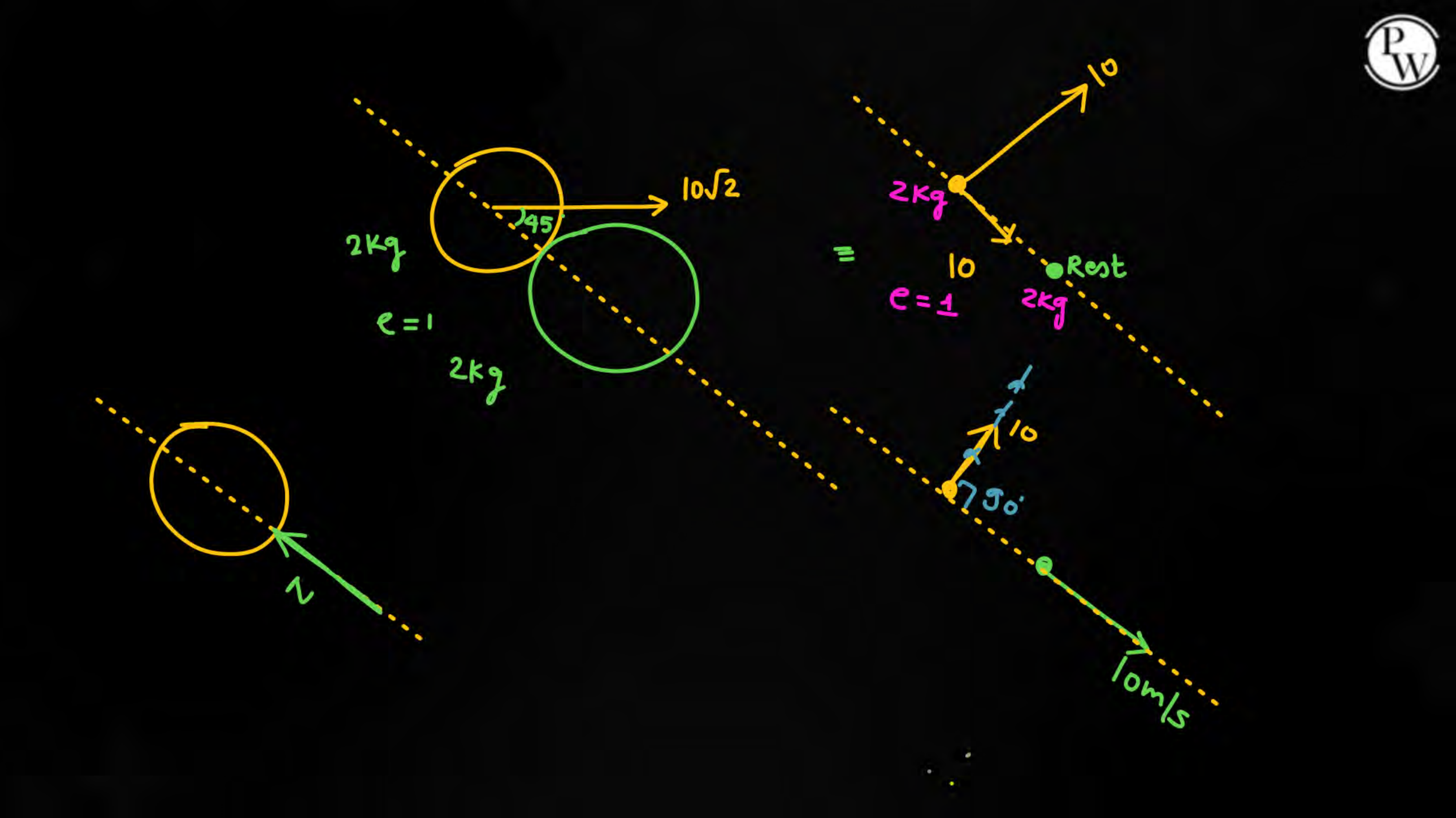
$$v_2 = 10$$

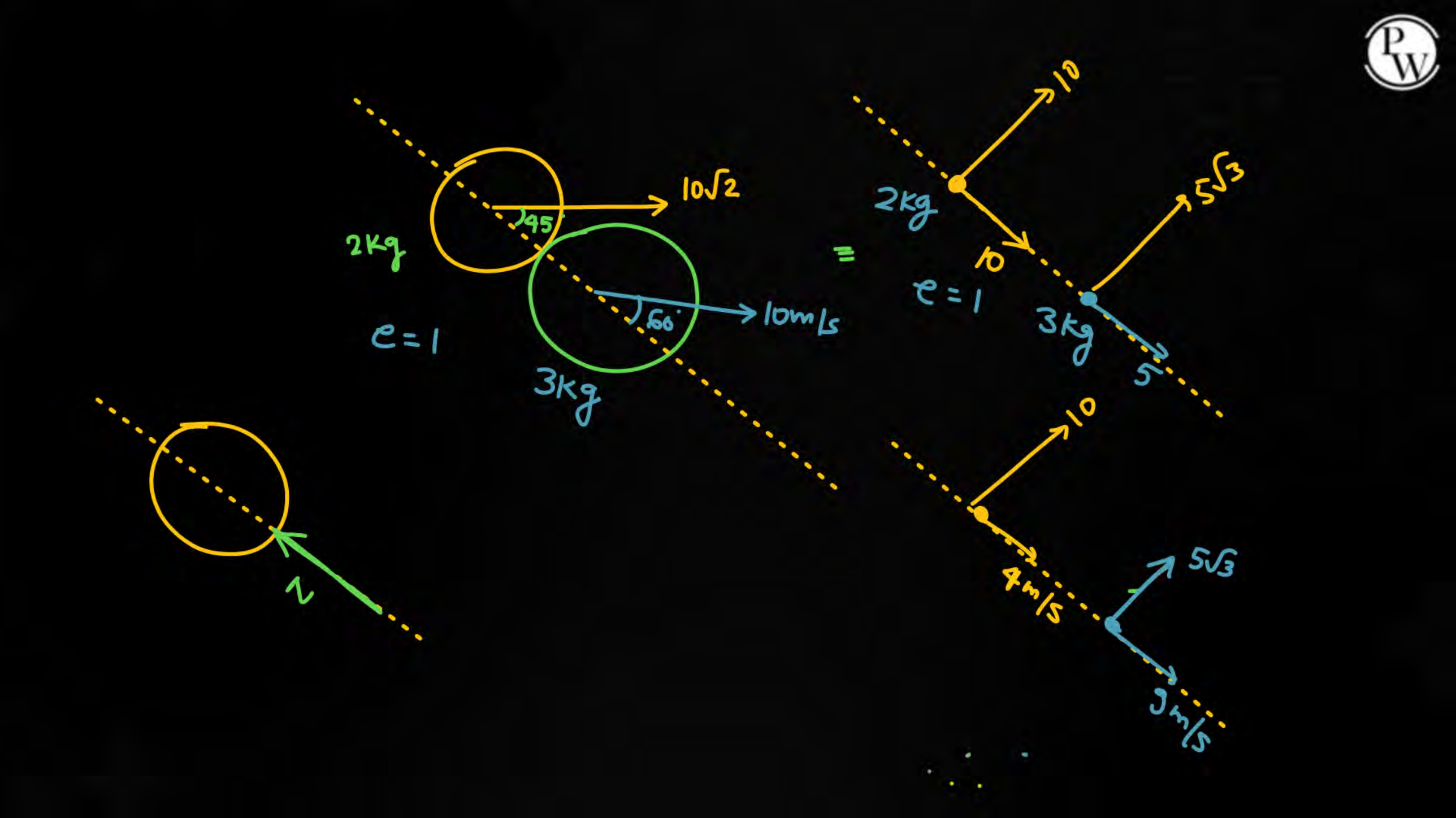
$$v_1 = 5$$

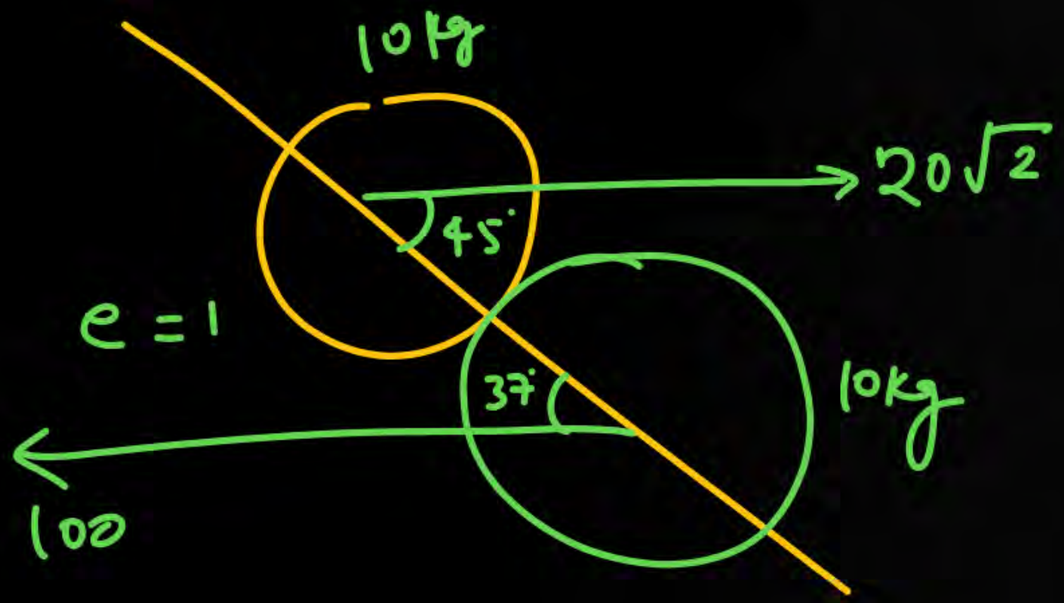
$$e = 1 = \frac{v_2 - v_1}{5}$$

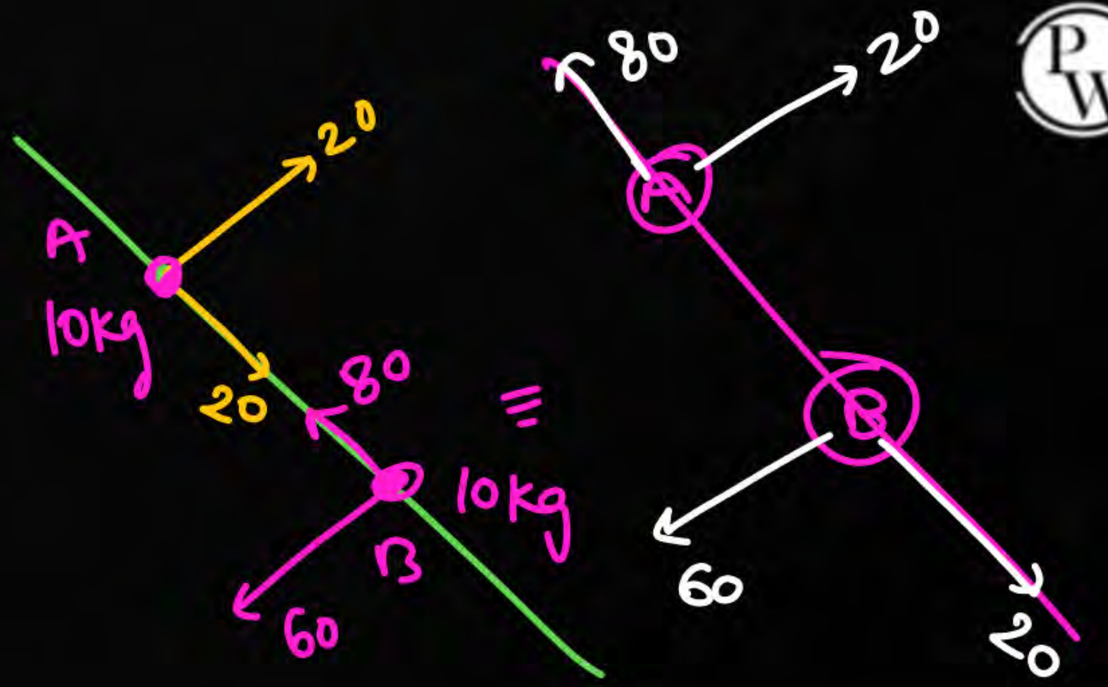
Q

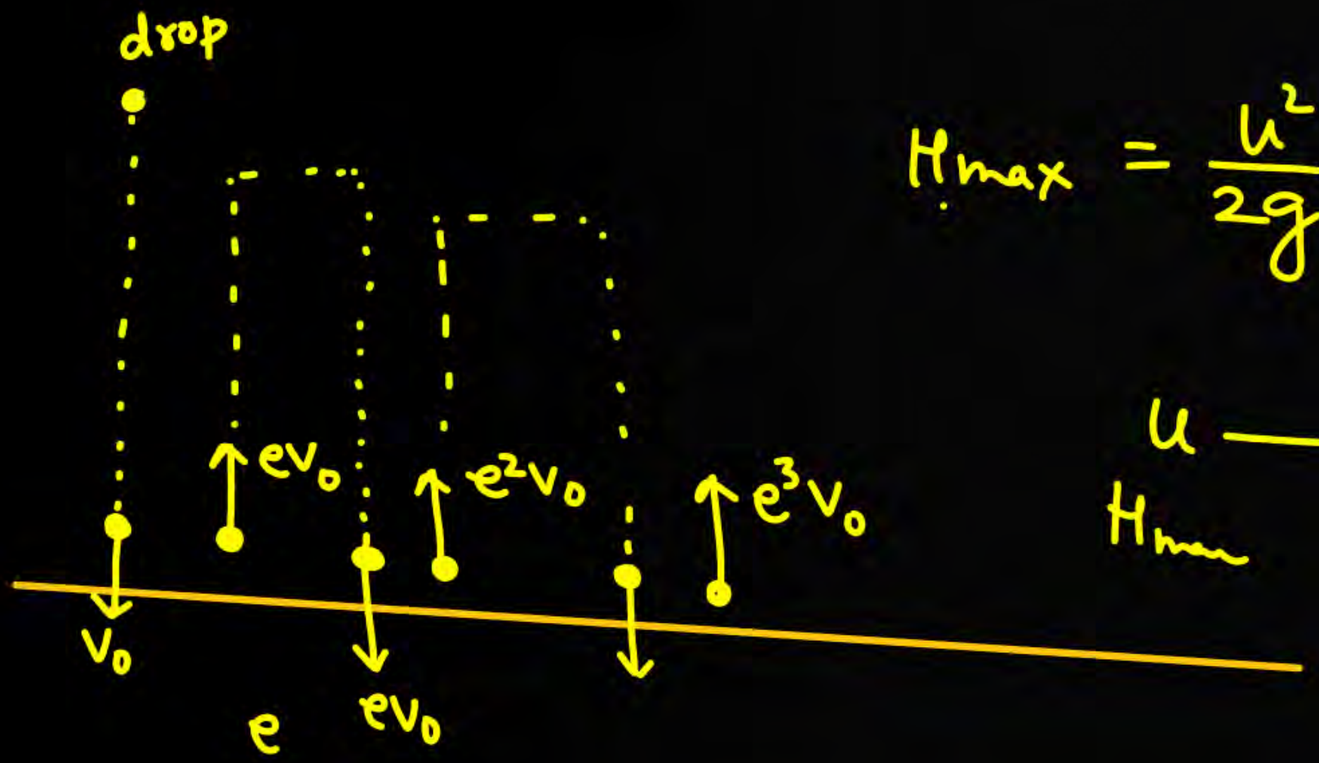



10√2
45
2kg
e=1
2kg
N
10
2kg
=
10
e=1
Rest
2kg
10
30
10m/s


2kg
45°
10√2
60°
10m/s
3kg
e = 1
≡
2kg
10
10
e = 1
3kg
5√3
5
N
10
4m/s
5√3
3m/s


10kg
$20\sqrt{2}$
45°
e = 1
37°
10kg
100


A
10kg
20
20
80
B
10kg
60
≡
80
20
A
B
60
20



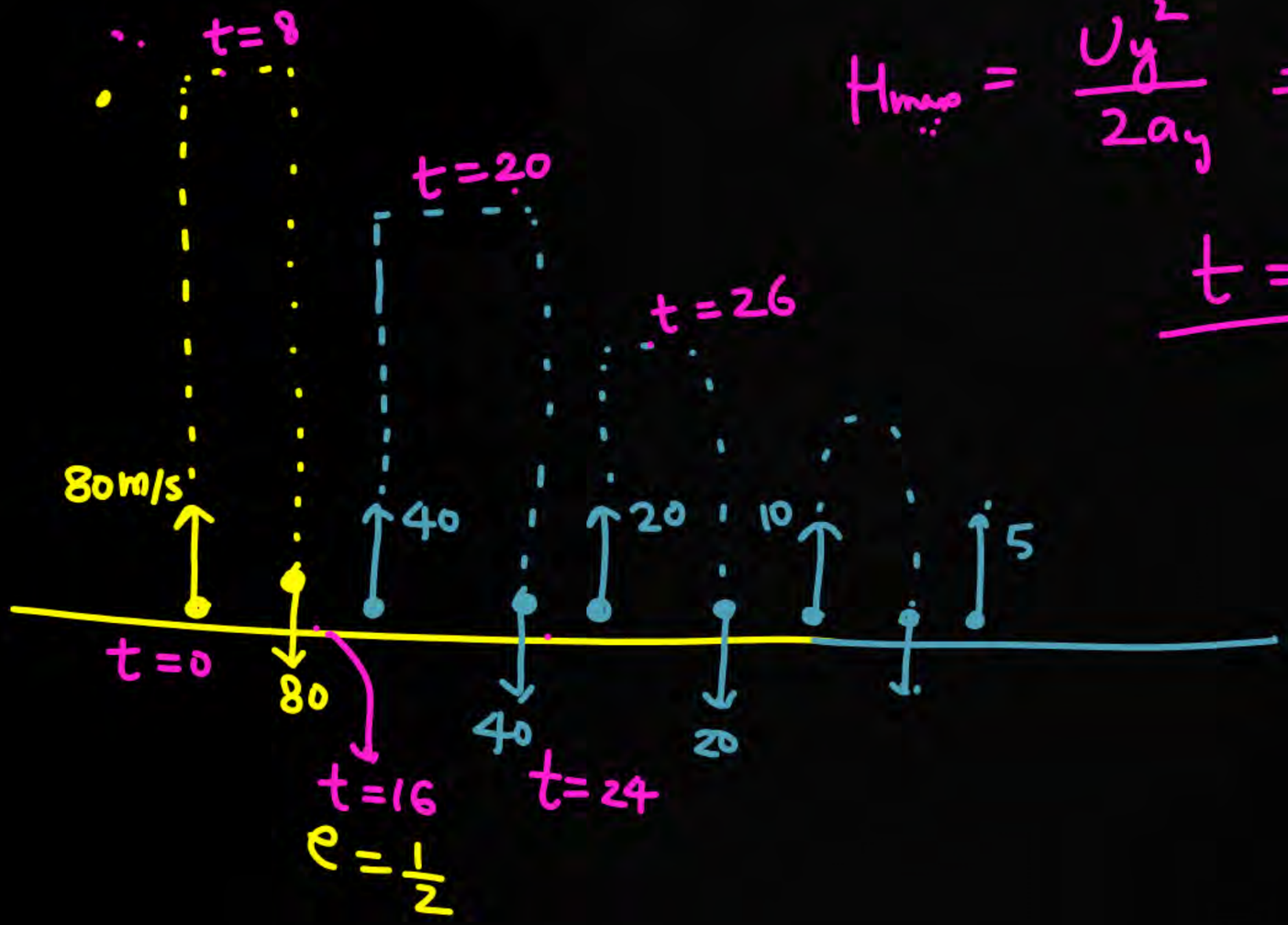
$$H_{max} = \frac{u^2}{2g}$$

$u \longrightarrow ue$

H_{max}

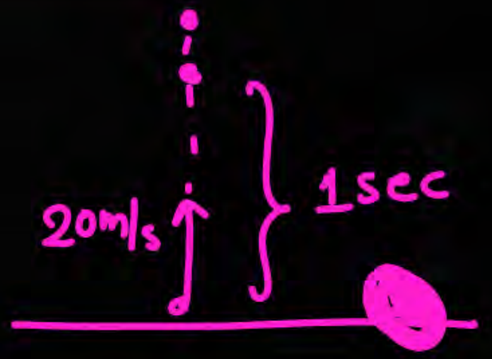
$H_0 \equiv e^2 H_0$ - - .

Q



$H_{max} = \frac{u_y^2}{2a_y} =$

t = 25 location

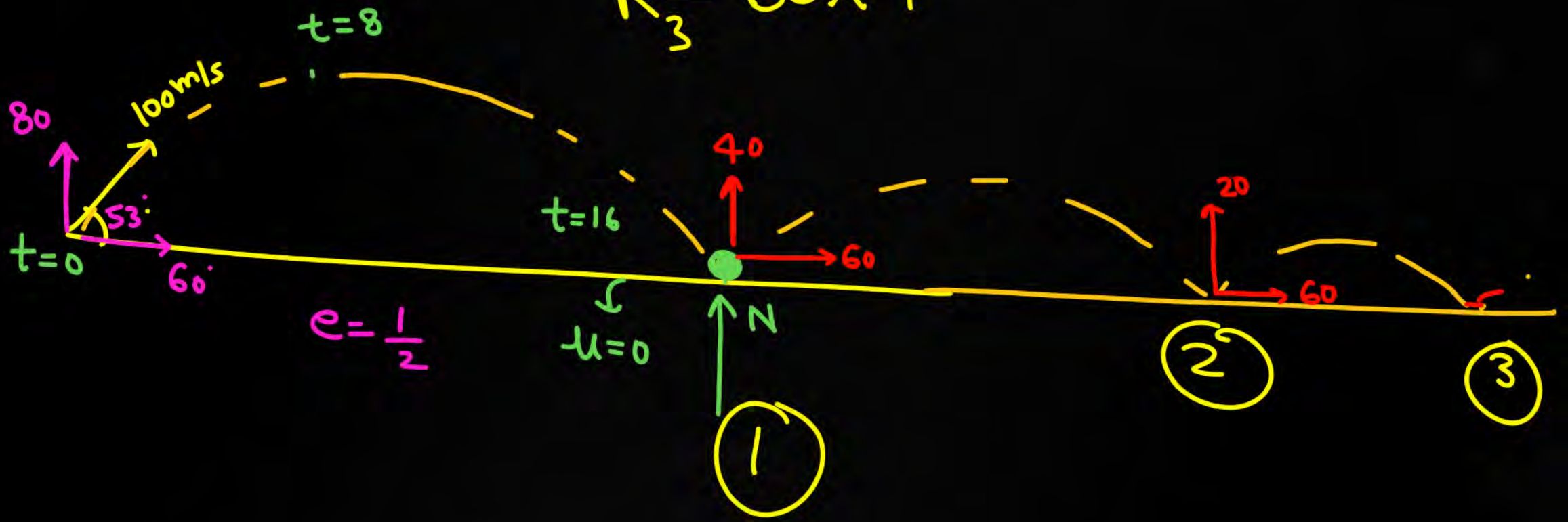


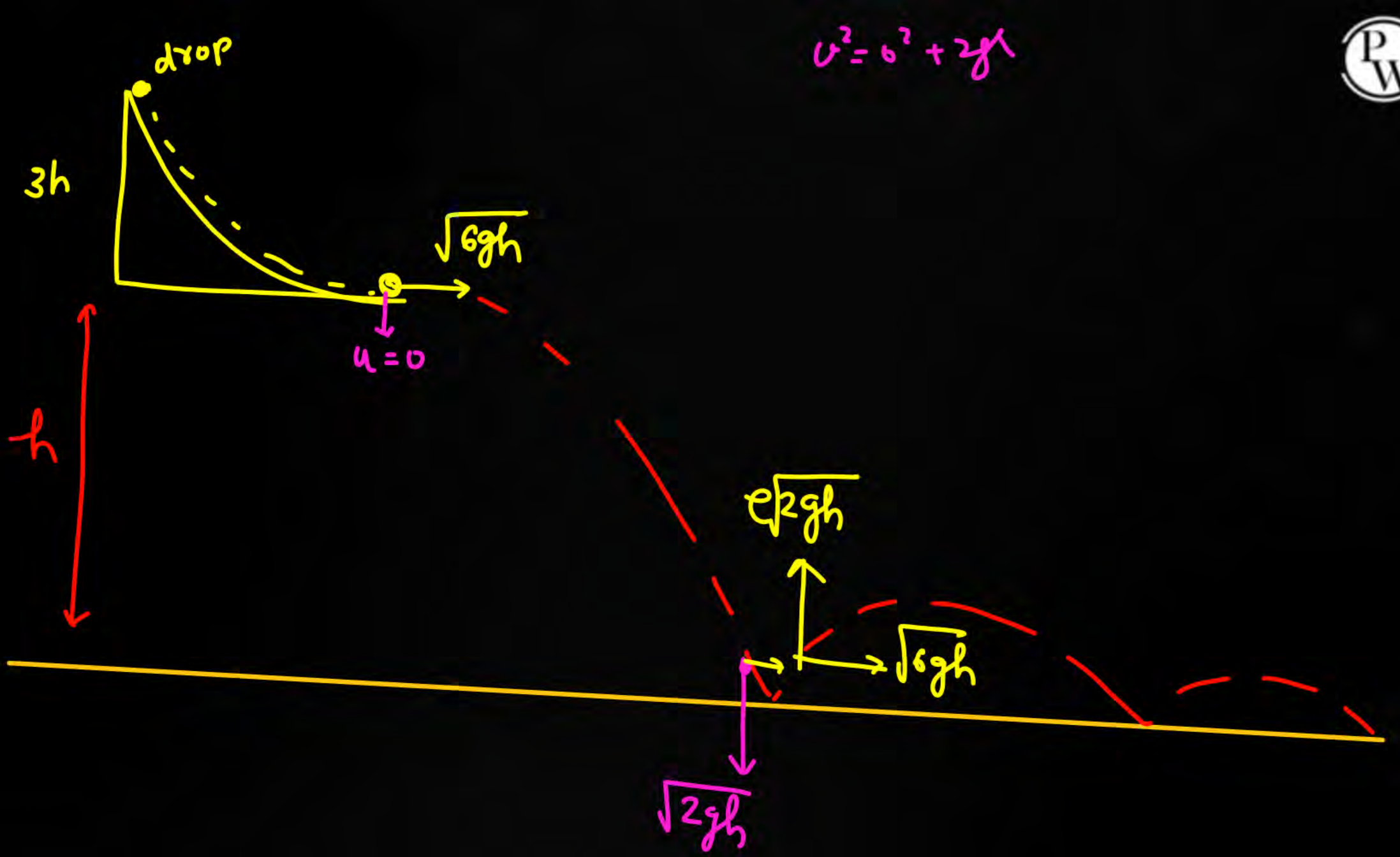
$y = 20 \times 1 - \frac{1}{2} \times 10 \times 1^2$

Where & when 3rd collision occur

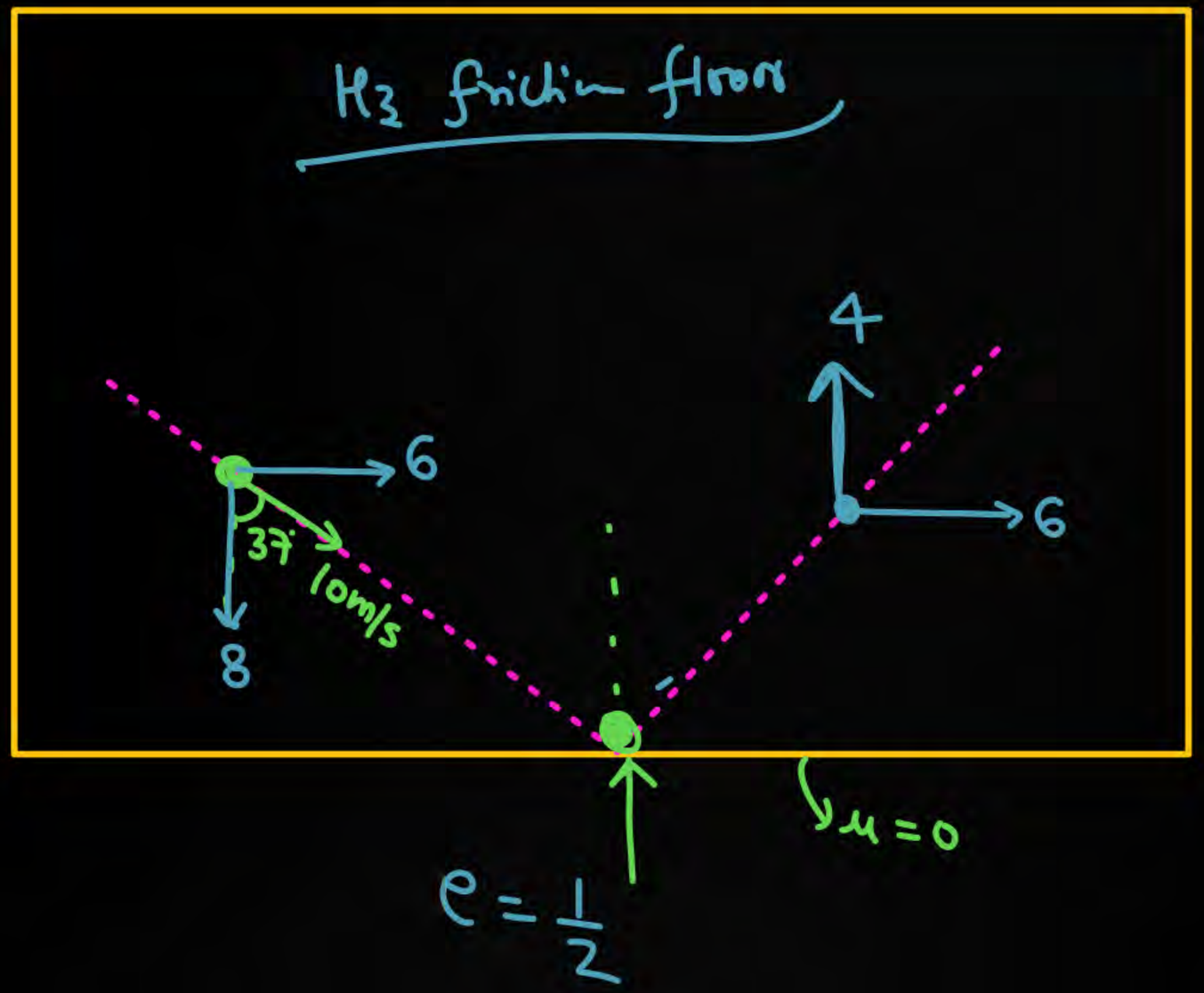
$T = 8+8+4+4+2+2$

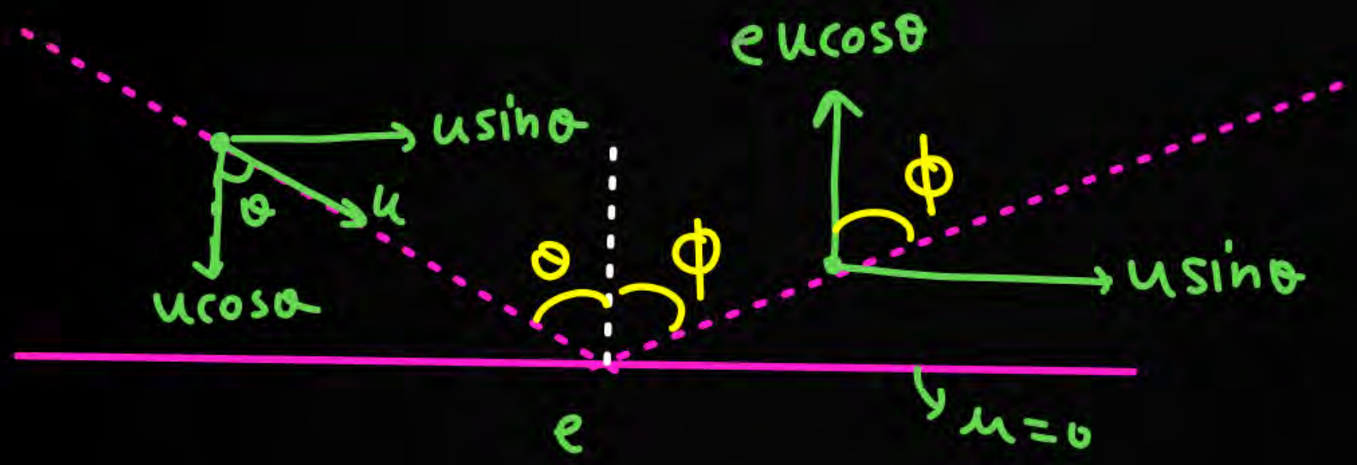
$R_3 = 60 \times T$





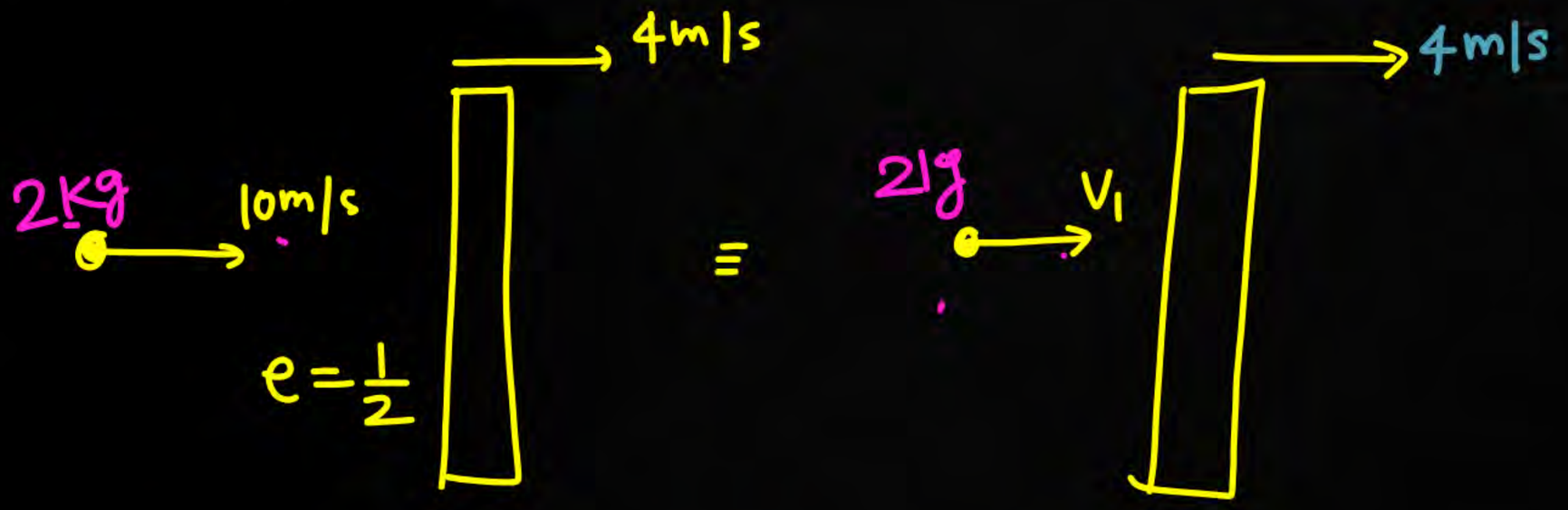
$v^2 = u^2 + 2gh$


H3 frictionless floor
6
37°
10m/s
8
4
6
e = 1/2
μ = 0



$$\tan\phi = \frac{u\sin\theta}{eu\cos\theta} = \frac{\tan\theta}{e}$$

$e = 1$, $\phi = \theta$



$P_i = 20$

$$e = \frac{1}{2} = \frac{4 - V_1}{6}$$

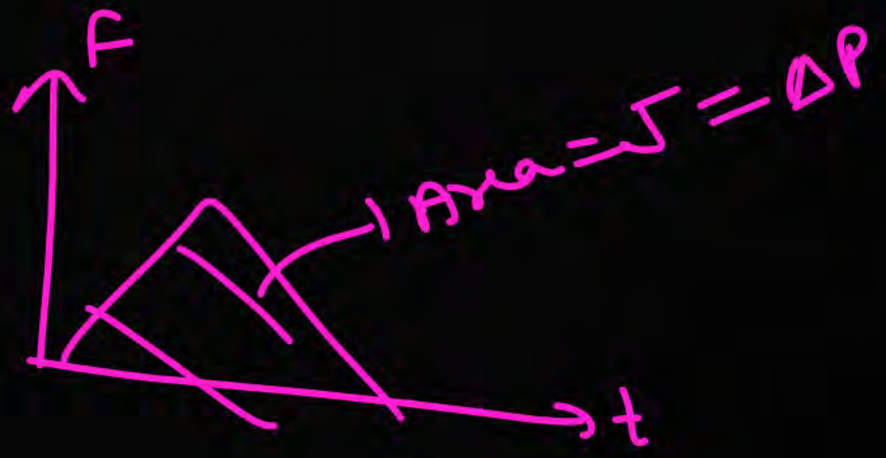
$$4 - V_1 = 3$$

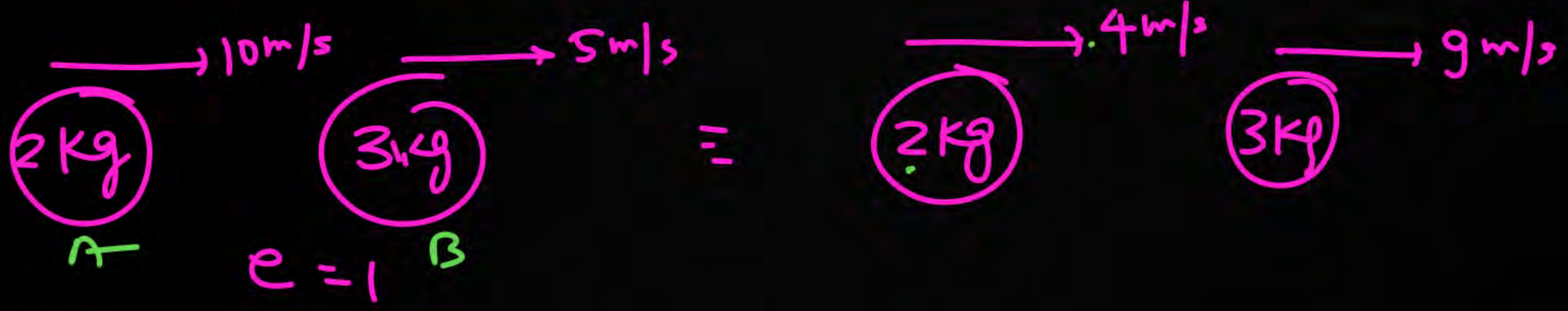
$$V_1 = 1$$

Impulse

$$\vec{J} = \int \vec{F}\,dt$$

$$= \Delta\vec{P}$$





(A) $\Rightarrow$ $\vec{P_i} = 20$

$\vec{P_f} = 8$

$\Delta\vec{P} = -12$

$\vec{d}_{com} = 0$

$m_1 \vec{d_1} + m_2 \vec{d_2} = 0$

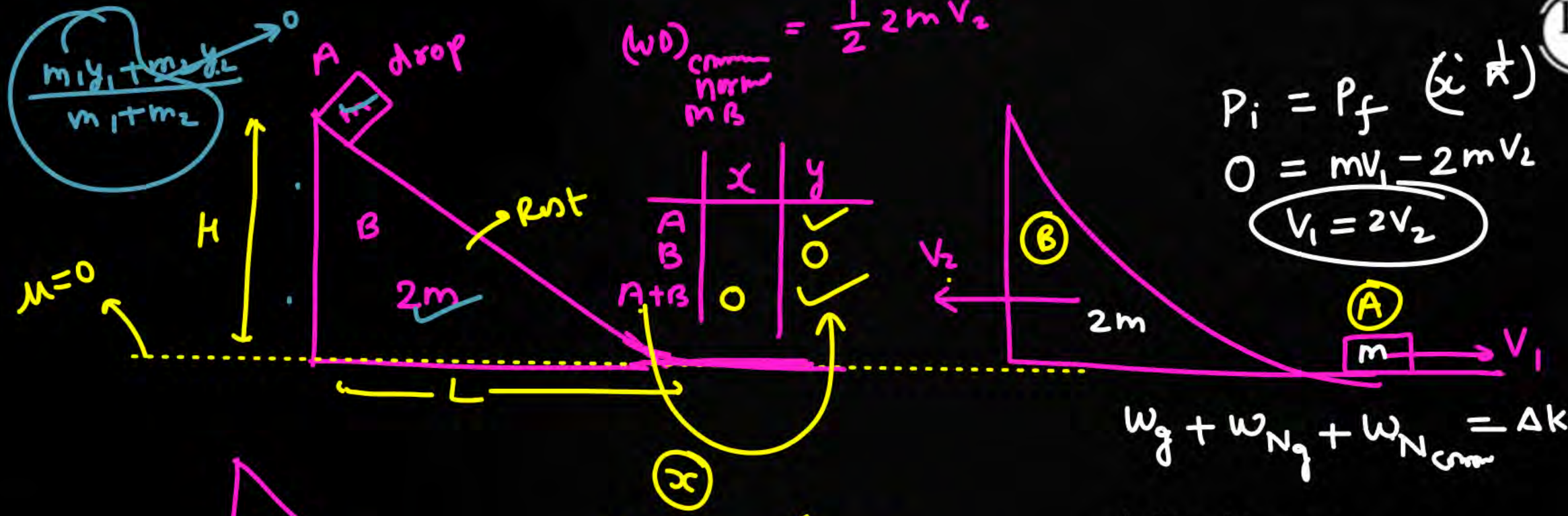
$m x + 2m(L+x) = 0$

$x = -\frac{2L}{3}$

man ≡ A → B

Q

$80x + 50(5+x) + 60x(10+x) = 0$



$$\frac{m_1y_1+m_2y_2}{m_1+m_2}$$

$$(WD)_{\substack{\text{conservative}\\ \text{forces}\\ \text{on } B}} = \frac{1}{2}2mV_2^2$$

$$P_i = P_f \quad (\text{in } x)$$

$$0 = mV_1 - 2mV_2$$

$$V_1 = 2V_2$$

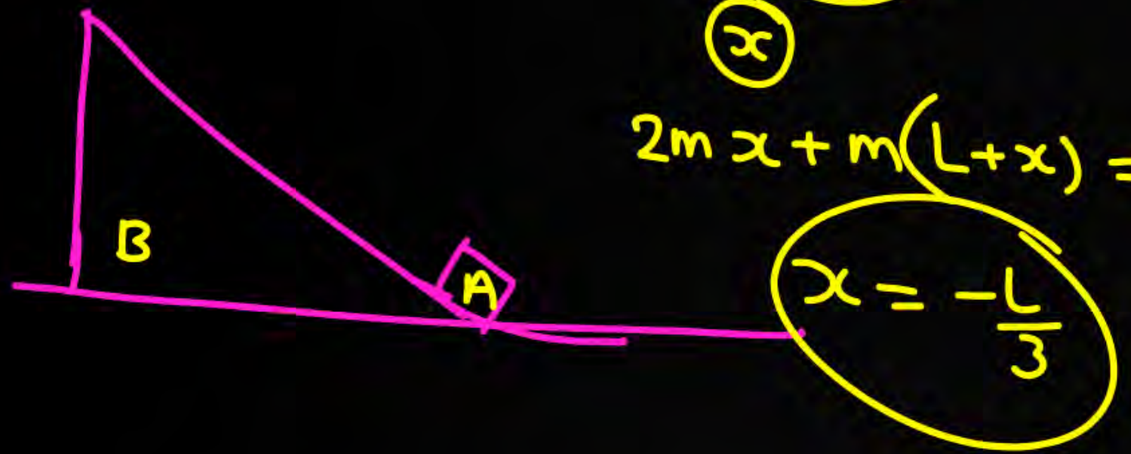
$$W_g + W_{N_g} + W_{N_{conv}} = \Delta K.E.$$

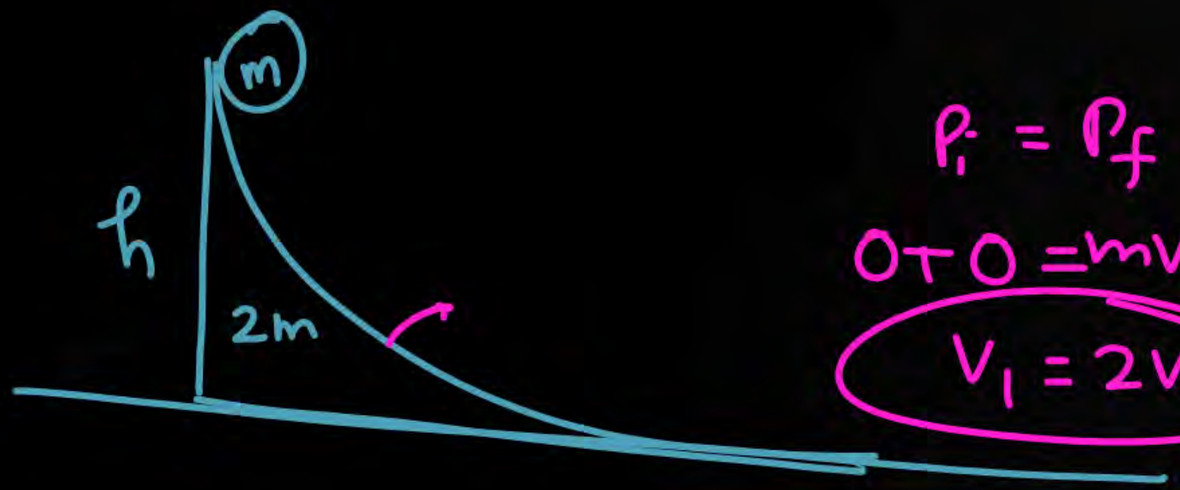
$$+mgH + 0 + 0 = \left(\frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2\right) - 0$$

(x)

$$2m\,x + m(L+x) = 0$$

$$x = -\frac{L}{3}$$

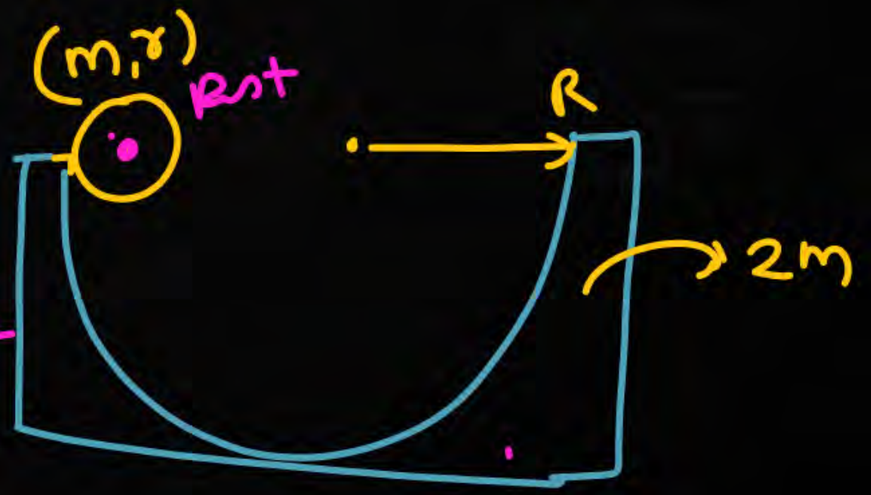


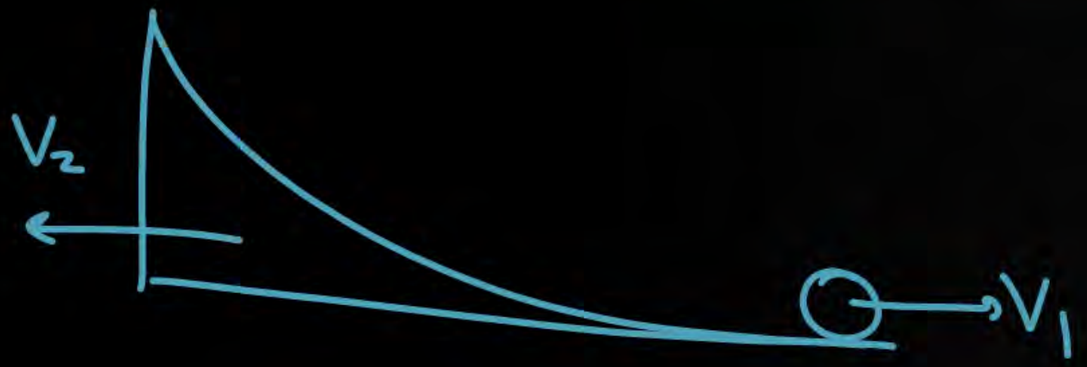


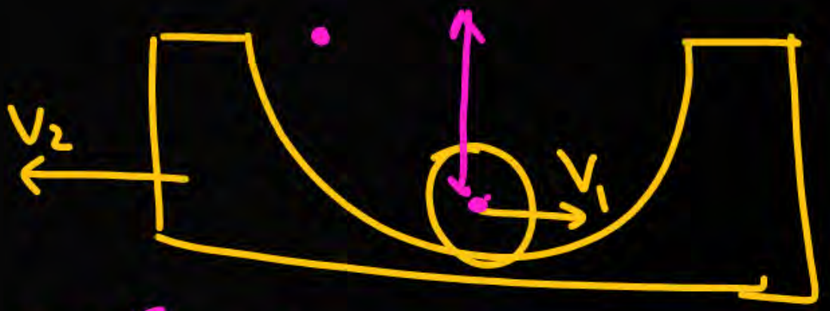
$$P_i = P_f$$

$$0 + 0 = mV_1 - 2mV_2$$

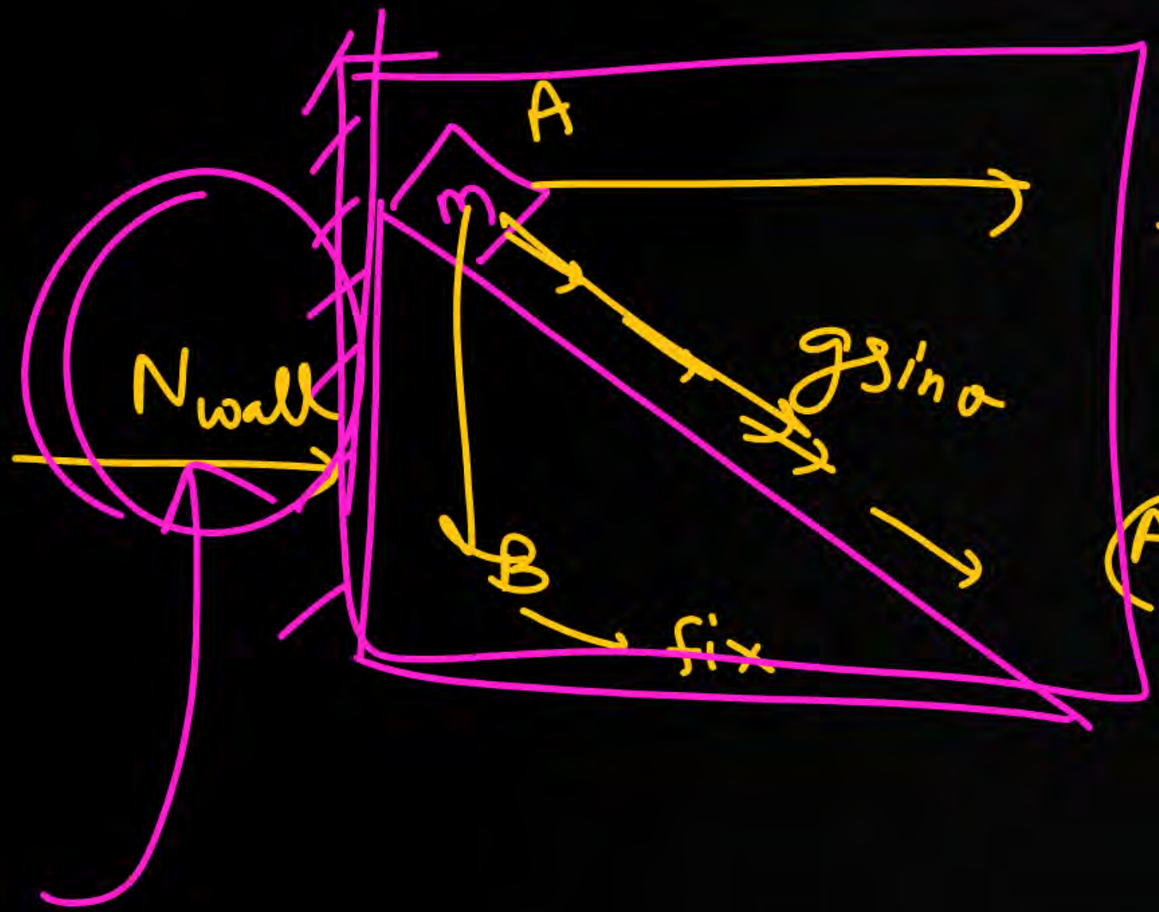
$$V_1 = 2V_2$$







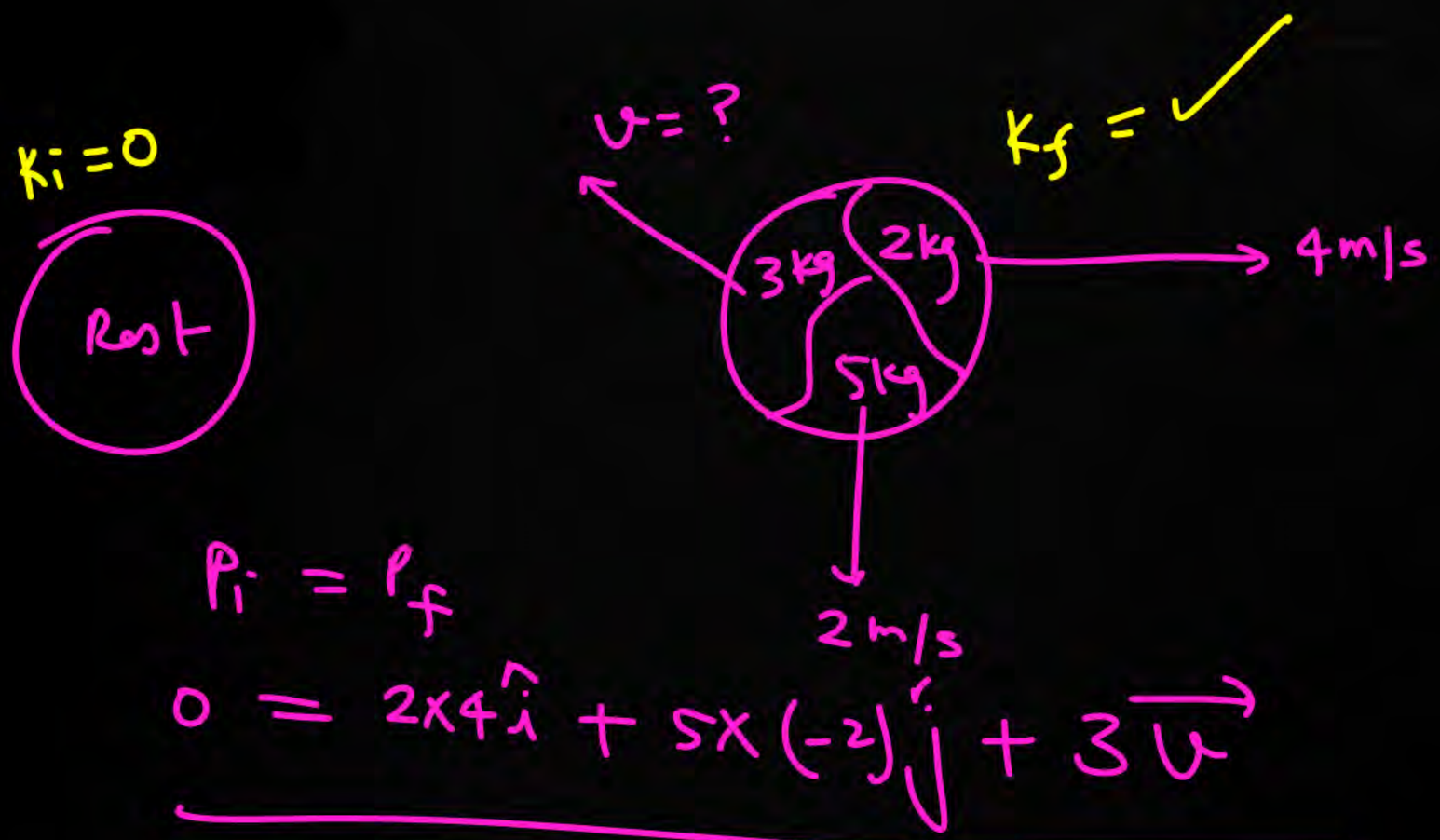
$$mg(R-r) = \frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2$$



	dx	dy	v_x	v_y	a_x	a_y
B	0	0	0	0	0	0
A	✓	✓	✓	✓	✓	✓
(A+B)	✓	✓	✓	✓	✓	✓

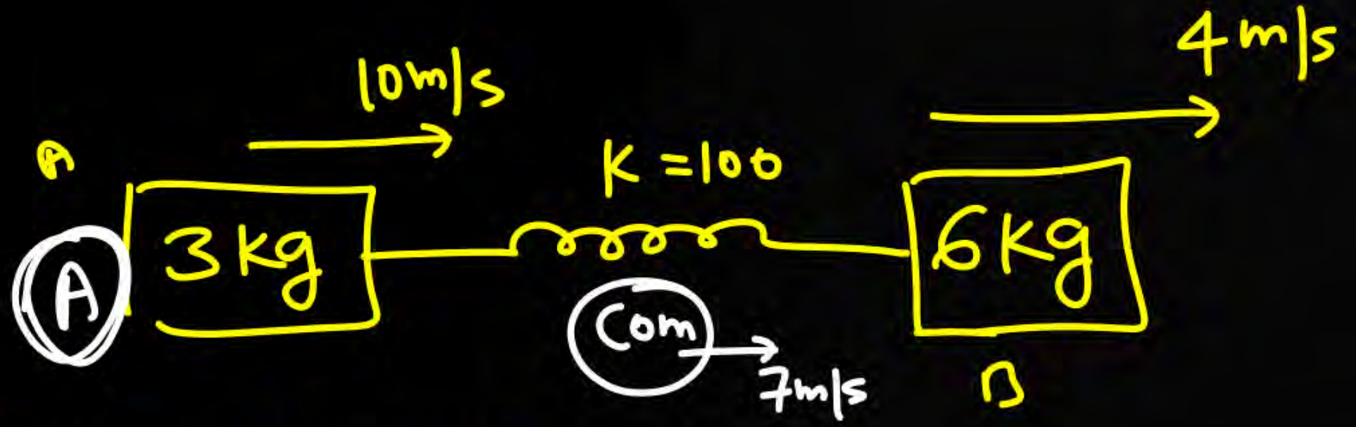
? 0

1



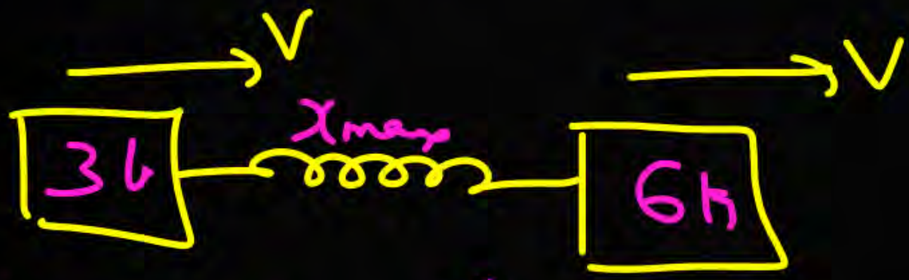
$$P_i = P_f$$

$$0 = 2 \times 4\hat{i} + 5 \times (-2)\hat{j} + 3\vec{v}$$



$$-\frac{1}{2}K\left(x_{max}^2 - 0^2\right) = K_f - K_i$$

① at max compress



$$30 + 24 = (3+6)v$$

$$54 = 9v$$

$$\boxed{v = 6}$$

$$v_{cm} = \frac{30+24}{3+6} = \frac{54}{9} = 6$$

THANK YOU